OPTIMAL CH-47 AND C-130

WORKLOAD BALANCE

THESIS

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THESIS

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Abstract

The Air Force can save thousands of dollars by reducing the number of blade hours on the CH-47 through finding an optimal mixture of CH-47s and C-130s to conduct current operations in Afghanistan and Iraq. Ultimately these savings will relieve maintenance operations for the CH-47 and lengthen the lifespan of the CH-47 airframe. Moreover, incorporating C-130s into the operations will reduce cargo transit time from supply depots.

This study looks at the involvement of the C-130 in CH-47 airlift operations to reduce CH-47 usage and increase supply efficiency. The research focus is narrowed to current airlift operations in Afghanistan and Iraq in the CENTCOM theater of operation. A mathematical representation of current CH-47 operations augmented with C-130s is the foundation of this research. Particularly, these operations in CENTCOM's area of operations are formulated as linear transportation problems using network mathematics.

The uniqueness this research offers entails modified scenarios of the transportation problem solved as an optimization model. AMC requires additional constraints to be augmented with the basic transportation linear model that pushes this application in new areas. In addition, the uncommon layout of supply depots to the specified receiving airfields in Afghanistan and Iraq provide an altogether new kind of transportation problem.

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Captain Dustin P. Jones, USAF

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I. Introduction

Background

The CH-47 network in U.S. Central Command's (CENTCOM's) area of operations is comprised of ten airfields. Two of these airfields are capable of handling C-130s. Additionally, one of these C-130 capable airfields acts as the central hub for the area of operations for Afghanistan and Iraq. Forty percent of CENTCOM's supply requirements for the CH-47 move out from the central hub. Another forty percent of requirements are flown into the central hub airfield. The remaining twenty percent of supplies and passengers are moved to and from other airfields that do not include the central hub. The C-130 capable airfields are utilized as standard CH-47 airfields; utilizing C-130s to perform the portion of CH-47 workload involving these airfields has potential to reduce CH-47 hours.

Problem Statement

The current OPS tempo and the austere terrain of Afghanistan and Iraq place a heavy burden on the CH-47 fleet. Currently the CH-47 schedule is built independently of the C-130 schedule in both Iraq and Afghanistan. Some of the intra-theater lift performed by the CH-47 fleet is to airfields that support C-130 operations. The potential exists for C-130s to relieve some of the CH-47 workload and save blade hours. The problem then becomes a question of optimizing the blending of CH-47 and C-130 routes.

Research Objectives

This research captures the underlying CH-47 network structures in both Iraq and Afghanistan. Additionally, it is desired to simulate flight times and cargo/passenger requirements at these locations. Air Mobility Command/Analysis, Assessment, and Lessons Learned Directorate (AMC/A9) can provide the CH-47 and C-130 flight schedules as well as the requirements that drove those schedules. Once the network is defined along with the requirements that drive the need for CH-47 and C-130 airlift assets, an optimization model is developed to minimize costs. Given AMC/A9 data and the optimization model, the following are research objectives:

a) Determine how many CH-47 blade hours can be saved
b) Determine what additional C-130 effort is required to garner these savings
c) Build a method to determine the correct C-130-CH-47 mix based on cargo and passenger requirements and the number of C-130 capable and non C-130 capable airfields.

Research Focus

This research centers on CH-47 transport requirements between the primary hub and several forward airfields in the CENTCOM area of operations. Historical data provided by Air Mobility Command indicates the usual CH-47 airlift requirements since the start of the Afghanistan and Iraq wars on terrorism. This data forms the foundation of research analysis as the movement of cargo and passengers is tracked from the central hub to various airfields.

Methodology.

The analysis is accomplished with the aid of an AMC optimization model that supports CH-47s and C-130s and a developed Microsoft Excel® compatible optimization model. A collection of flight schedules flown to Afghanistan and Iraq over a specified time period is the data needed to execute this research analysis. The essential optimization parameters include the central and destination airfield locations, the number of passenger requirements, the amount of cargo requirements, and travel time between airfields. AMC/A9 provided the bulk of the data sources.

Assumptions/Limitations.

The developed optimization model is unique to this transportation scenario in Afghanistan and Iraq. Moreover, the parameters utilized are specific to the CH-47 and the C-130. It is assumed that there are available C-130s to augment CH-47 operations as well as maintain a similar operation tempo. Based on the reduced number of CH-47s utilized, the number of blade hours saved is estimated from the equivalent workload amount performed by the C-130s that would otherwise be performed by CH-47s.

Implications.

The goal of this research project is to present Air Force and Army leadership with an efficient, combined employment of joint airlift assets. Results shape the size of airlift capabilities needed to satisfy CENTCOM's requirements and the allocation of those capabilities. CH-47 savings become tangible in the form of decreased funding and maintenance resources, reduced airframe and blade wear, and retained CH-47 availability.

Research Overview

This investigative study commences with an in-depth review of research linear formulations and heuristic approaches on spoke-and-hub networks. Several integer programming formulations are presented in chronological order based on the date published. These formulations consist of p-hub median/center problems, p-hub covering problems, and capacitated versus uncapacitated hub location problems. The heuristic literature review include topics on the Genetic Algorithm, Tabu Search, and Lagrangean heuristic. The research then shifts to a review on Vehicle-Routing Problems (VRP) and provides several helpful formulations applicable to balance the CH-47 workload with support from the C-130. An investigation on application of the above areas to military airlift concludes the literature review, which is presented in Chapter 2.

In Chapter 3, the insight garnered from the research studies and data provided by U.S. Transportation Command (USTRANSCOM) is used to develop a methodology to find a balance in the CH-47's workload. Capturing the CH-47 scenario and network is the first step towards developing a mixed integer linear program in the LINGO ® software environment. The program development is based on the constraints established by the sponsor and the information gleaned from CH-47 sortie records from the Iraq theater from 2009 to 2010.

Results and analysis presented in Chapter 4, and conclusions presented in Chapter 5 are based on a demonstrative scenario using actual aircraft parameters, and flying hour costs from DoD records. The scenario incorporates the same number of hubs and available CH-47 aircraft currently employed in the Iraq theater. A similar daily cargo requirement is generated and a baseline CH-47 workload is captured. Analysis

performed is handled by incrementally inserting C-130 aircraft into the scenario and evaluating the reduction in needed CH-47 flying hours, and impact on daily cost.

II. Literature Review

Introduction

The geographical scenario presented by the research problem at hand can unquestionably be represented as a “hub-and-spoke” network design. This design is very common in multiple industries such as network communications, package delivery, and passenger airlines. Despite the common network design, these hub-and-spoke problems belong to the NP-hard class of problems (Abdinnour-Helm and Venkataramanan, 1998). In order to tackle these difficult problems, this literature review encompasses previous research on solving hub-location problems looking at heuristic and integer programming approaches. Additionally, a close look at vehicle-routing problems is included in the following review as they are closely related, if not all encompassing, to hub-location problems. Both problems types can be, and indeed have been, used to determine how to optimize vehicle coverage, but the key difference is hub-location problems are aimed to optimize flow throughout a network whereas vehicle-routing problems tend to optimize coverage. This research optimizes both passenger and cargo flow through the network as well as ensures that there are enough air assets to cover the locations that have requirements. The last portion of this literature review covers military application and research of solutions to hub-and-spoke network designs.

Hub-and-spoke Research

This part of the review looks at several heuristic and mathematical programming methods for solving hub-location problems. Generally, the solution techniques developed can be applied to scenarios containing multiple hubs. Some research

discussed places emphasis on uncapacitated versus capacitated hubs such that the flow may be limited in the latter case. It is important to note that the bulk of the research reviewed treats the location and number of hubs as variables which significantly increases the difficulty in the problem to be solved. However, the Afghanistan and Iraq hubs unique to this research problem are known and fixed. This leads to the research questions that need to be answered:

- Which spokes are linked to each hub?
- How much of the requirements can be flown out of and between the hubs (capacities)?

The approaches previously researched and developed can be utilized to help answer these questions.

Integer Programming Formulations.

This section of the literature review provides the fundamental background related to the intended methodology to be implemented in formulating the CH-47 hub-and-spoke networks in Afghanistan and Iraq. The takeaways from the chosen literature below are: how to formulate a generic hub-and-spoke network as an integer, preferably, linear program; how to account for flow requirements; and the assignment of resources needed to deliver the requirements from hub to destination. This last aspect is particularly significant to the research as the goal is to reduce CH-47 blade hours by incorporating other resources (C-130s) to deliver requirements. Once the network is captured and formulated as a linear program, the only aspects which change are the daily flow requirements and resources required.

The first model discussed is O'Kelly's quadratic integer program through the perspective of Aykin. While this model is formulated as a quadratic program, it sheds light on concepts that can be readily applied to the linear hub location program. The general problem is to find the locations of the hub facilities and the node assignments that minimize the total transportation cost (Aykin, 1990). Again the research in this thesis is not interested in determining hub locations as they are fixed and known. What is of interest is the aircraft assignment methodology for the hubs. In relation to the research, it is desired to find an assignment formulation for multiple airlift resources. This formulation resembles the transportation problem formulation.

Variables of interest in O'Kelly's quadratic integer program are the hub locations and which nodes are assigned to them. Given *n* interacting nodes, flows between pairs of nodes denoted by W_{ij}, the transportation cost C_{ij} of a unit of flow between nodes *i* and *j*, and the number of hub facilities *p* to be located (Aykin, 1990), the following model is developed to solve for node assignment variable X_{ik}:

$$Minimize\ Z = \sum_i \sum_j \sum_k \sum_m W_{ij}\left[X_{ik}X_{jm}\left(C_{ik} + aC_{km} + C_{jm}\right)\right]$$

$$subject\ to\ \sum_i X_{ik} \leq (n-p+1)X_{kk}\ for\ all\ k,$$

$$\sum_k X_{ik} = 1\ for\ all\ i,$$

$$\sum_k X_{kk} = p,$$

$$0 \leq X_{ik} \leq 1\ and\ integer\ for\ all\ i\ and\ k$$

Figure 1 - O'Kelly's formulated model (Aykin, 1990)

In the above formulation, k represents the location of hub, but α was not specifically defined.

Campbell (1994) presents four additional integer programming formulations fitting this problem. Two of these formulations, the hub median and uncapacitated hub location problems, have been cited and applied by researchers. The last two formulations introduced by Campbell model the discrete hub center and hub covering problems. In regards to discrete hub location problems, the following five items are considered to be given, or known:

1) n demand locations
2) r potential hub locations
3) The flow for the n^2 demand location pairs
4) The per unit cost between all location pairs, and
5) The hub-to-hub discount factor α (Campbell, 1994).

The first hub-and-spoke model discussed is the p-hub median problem. Its objective is to minimize total cost of flow units and is considered to be particularly important to the airline industry. Before the model is shown below, the following variables and parameters have been defined for the set of hub location problems: X_{ijkm} is the fraction of flow from location i to location j that is routed via hubs at locations k and m in that order, Y_k is a (1,0) variable if location k is a hub, Z_{ik} is a (1,0) variable if location i is allocated to the hub at location k, W_{ij} is the flow from location i to location j, and C_{ij} is the standard cost per unit from location i to j (Campbell, 1994). Given these definitions, the below p-hub median problem is formulated as a linear program below:

$$Minimize \sum_i \sum_j \sum_k \sum_m W_{ij} X_{ijkm} C_{ijkm}$$

$$subject\ to \sum_k Y_k = p,$$

$$0 \leq Y_k \leq 1\ and\ integer\ for\ all\ k,$$

$$0 \leq X_{ijkm} \leq 1\ for\ all\ i, j, k, m,$$

$$\sum_k \sum_m X_{ijkm} = 1\ for\ all\ i, j,$$

$$X_{ijkm} \leq Y_k\ for\ all\ i, j, k, m,$$

$$X_{ijkm} \leq Y_m\ for\ all\ i, j, k, m$$

Figure 2 – Campbell's (1994) formulated *p*-hub median problem

The next formulation is that of the uncapacitated hub location problem. The underlying difference here is that the number of hubs is an unknown. Consequently, since the number of hubs is unknown, there is an associated fixed cost for generating each hub. As such, Campbell defines the parameter F_k as the fixed cost of establishing a facility at location k (Campbell, 2004). This model is formulated as shown below:

$$Minimize \sum_i \sum_j \sum_k \sum_m W_{ij} X_{ijkm} C_{ijkm} + \sum_k F_k Y_k$$

$$subject\ to\ 0 \leq Y_k \leq 1\ and\ integer\ for\ all\ k,$$

$$0 \leq X_{ijkm} \leq 1\ for\ all\ i, j, k, m,$$

$$\sum_k \sum_m X_{ijkm} = 1\ for\ all\ i, j,$$

$$X_{ijkm} \leq Y_k\ for\ all\ i, j, k, m,$$

$$X_{ijkm} \leq Y_m\ for\ all\ i, j, k, m$$

Figure 3 – Campbell's (1994) formulated uncapacitated hub location problem

For the hub center problem, the goal is to minimize the maximum cost for any origin-destination pair of nodes. The hub center itself can be a single hub or consist of a collection of hubs. This type of hub center is important for a hub involving perishable or time sensitive items, in which cost refers to time, α is a time discount factor due to higher speed on the inter-hub links, and the maximum time from an origin-to-destination is of interest (Campbell, 1994). His basic *p*-hub center problem formulation is presented below:

$$Minimize \underset{i,j,k,m}{Maximum}\{X_{ijkm}C_{ijkm}\}$$

$$subject\ to\ \sum_k Y_k = p,$$

$$0 \leq Y_k \leq 1\ and\ integer\ for\ all\ k,$$

$$\sum_k \sum_m X_{ijkm} = 1\ for\ all\ i, j,$$

$$X_{ijkm} \leq Y_k\ for\ all\ i, j, k, m,$$

$$X_{ijkm} \leq Y_m\ for\ all\ i, j, k, m,$$

$$0 \leq X_{ijkm} \leq 1\ for\ all\ i, j, k, m$$

Figure 4 – Campbell's (1994) formulated *p*-hub center problem

Additionally, several variants of this formulation were developed by Campbell. These formulations can: minimize the maximum cost for movement on any single link, account for flow thresholds for spoke nodes, and take the form of a quadratic program (Campbell, 1994).

The last hub location problem type developed is the hub covering problems, which Campbell describes as being analogous to facility covering problems. It is desired to determine locations for the hubs such that they can satisfy all destination node demands while minimizing the cost of the hubs. In this formulation, the objective function is merely the hub cost piece from the uncapacitated hub location formulation.

$$Minimize \sum_{k} F_k Y_k$$

$$subject\ to\ 0 \leq Y_k \leq 1\ and\ integer\ for\ all\ k,$$

$$X_{ijkm} \leq Y_k\ for\ all\ i, j, k, m,$$

$$X_{ijkm} \leq Y_m\ for\ all\ i, j, k, m,$$

$$\sum_{k} \sum_{m} V_{ijkm} X_{ijkm} \geq 1\ for\ all\ i, j$$

Figure 5 – Campbell's (1994) formulated hub covering problem

Aykin goes on to confirm the applicability of the hub-and-spoke network design to scenarios such as those posed by the CH-47 blade wear dilemma by indicating that one of the successful applications of the hubbing concept is found in air passenger/cargo transportation (Aykin, 1995). He examines network designs in which it is possible to flow units in a network from demand point to another demand point (i.e. nonstop travel) or where flow is channeled through hubs before reaching the final demand point (transient travel). The goal of these problems is to minimize total transportation costs through the network by determining an optimal mix of service types and hub locations (Aykin, 1995). The general hub location and routing problem formulated as a mathematical model accounts for an unknown number of hubs and whether direct or

indirect flow is utilized in the network. The following variables and parameters for such a mode are defined as:

- X_{ij} is a binary variable in which one is assigned if flows from *i* to *j* are shipped directly
- X_{iktj} is a binary variable in which one is assigned if flows from *i* to *j* are shipped with the routing *i* to hub *k* to hub *t* to *j*
- W_{ij} is the amount of flows from demand point *i* to demand point *j*
- O_{ij} is a binary variable in which one is assigned if the pair (*i*, *j*) is an element within the set of routes where nonstop service is permitted
- *d*(P, Q) is the distance between nodes P and Q
- and c_{ij} is the associated cost parameter (Aykin, 1995)

Given these definitions, Aykin's general model formulation of the problem is shown below:

$$Minimize \sum_i \sum_j O_{ij} W_{ij} c_{ij} d\left(P_i, P_j\right) X_{ij}$$

$$+ \sum_i \sum_k \sum_t \sum_j W_{ij} \left(a_1 c_{ik} d(P_i, Q_k) + a c_{kt} d(Q_k, Q_t) + a_2 c_{tj} d\left(Q_t, P_j\right) \right) X_{iktj}$$

$$subject\ to\ O_{ij} X_{ij} + \sum_k \sum_t X_{iktj} = 1\ for\ all\ i, j,$$

$$Q_k \in E^2\ for\ all\ k, X_{ij}, X_{iktj} \in [0,1]\ and\ integer\ for\ all\ i, k, t, j$$

Figure 6 – Aykin's (1995) general hub location and routing problem

The author goes on to indicate that for known hub locations, the above formulation can be reduced to a decision model that only accounts for service types:

$$Minimize \sum_i \sum_j W_{ij} C_{ij} O_{ij} X_{ij} + \sum_i \sum_k \sum_t \sum_j W_{ij} C_{iktj} X_{iktj}$$

$$subject\ to\ O_{ij} X_{ij} + \sum_k \sum_t X_{iktj} = 1\ for\ all\ i, j,$$

$$X_{ij}\ and\ X_{iktj} \in [0,1]\ and\ integer\ for\ all\ i, k, t, j$$

Figure 7 – Aykin's (1995) formulation for known hub location and routing problem

Five methods are proposed in order to find a starting solution to these problems. The first is to randomly choose initial hub locations. Another method is to randomly select a number of demand nodes to act as the beginning hub locations. As a third method, Aykin describes a 'Drop Solution' in which every demand node is assumed to be a hub. Each hub is then evaluated to determine the increase in the objective function value if it is closed; this is done by removing the hub locations one at a time (Aykin, 1995). Additionally, he proposes a drop and interchange method which incorporates obtaining an initial solution from the drop technique, but then proceeds to interchange the initially selected hubs with non-hub demand nodes to see if better objective function values can be obtained. The last suggested method is enumeration, but it is highly discouraged for use in expansive networks.

Ernst and Krishnamoorthy provide a similar, but altered formulation of the p-hub median problem that was presented earlier in Campbell's research and model development. The primary alteration lies in the objective function formulation. Campbell's is comprised of a single piece which accounts for flow and cost between nodes. Ernst's and Krishnammorthy's formulation consists of two distinct parts: the first piece captures the flow and costs from hub to demand node, and vice-versa, and the

second piece accounts for transfer costs between hubs, as applicable. For an uncapacitated single allocation problem, the authors present the following mixed integer linear program:

$$Min \sum_{i\in N}\sum_{k\in N} d_{ik}Z_{ik}(\chi O_i + \delta D_i) + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha d_{kl}Y_{kl}^{i}$$

$$s.t. \sum_{k\in N} Z_{kk} = p,$$

$$\sum_{k\in N} Z_{ik} = 1, \forall i \in N,$$

$$Z_{ik} \le Z_{kk}, \forall i, k \in N,$$

$$\sum_{l\in N} Y_{kl}^{i} - \sum_{l\in N} Y_{lk}^{i} = O_i Z_{ik} - \sum_{j\in N} W_{ij}Z_{ij} \ \forall i, k \in N,$$

$$Z_{ik} \in \{0,1\} \ \forall i, k \in N,$$

$$Y_{kl}^{i} \ge 0 \ \forall i, k, l \in N$$

Figure 8 – Ernst and Krishnmoorthy (1998) formulated *p*-hub median problem

They also present an uncapacitated multiple allocation *p*-hub median problem formulation where the model accounts for varying flow depending on the direction between two nodes (to include hubs as well). This particular formulation could be very useful in developing the unique flow constraints relative to the CH-47 problem as cargo and PAX requirements will vary depending on the air assets being employed. Ernst's and Krishnamoorthy's formulation is:

$$Min \sum_{i\in N}\left[\sum_{k\in N} \chi d_{ik}Z_{ik} + \sum_{k\in N}\sum_{l\in N} \alpha d_{kl}Y_{kl}^{i} + \sum_{l\in N}\sum_{j\in N} \delta d_{lj}X_{lj}^{i}\right]$$

$$s.t. \sum_{k \in N} H_k = p,$$

$$\sum_{k \in N} Z_{ik} = O_i, \forall i \in N,$$

$$\sum_{k \in N} X_{kl}^i = W_{ij}, \forall i, j \in N,$$

$$\sum_{l \in N} Y_{kl}^i + \sum_{j \in N} X_{kj}^i - \sum_{l \in N} Y_{lk}^i - Z_{ik} = 0, \forall i, k \in N,$$

$$Z_{ik} \leq O_i H_k, \forall i, k \in N,$$

$$X_{lj}^i \leq W_{ij} H_l, \forall i, j, l \in N,$$

$$X_{lj}^i, Y_{kl}^i, Z_{ik} \geq 0, \forall i, j, k, l \in N,$$

$$H_k \in \{0,1\} \; \forall k \in N$$

Figure 9 – Ernst and Krishnmoorthy (1998) formulated uncapacitated multiple allocation *p*-hub median problem

Recent research papers generally utilized the above hub-and-spoke formulations as a foundation for further model variants. For example, de Camargo *et al.* introduced a model formulation for multiple allocation hub location problems that accounts for flow congestion. This model utilizes a foundation formulation that largely resembles the research of Campbell and Ernst and Krishnamoorthy. In de Camargo’s research, the team defines a function that assesses hub loading and distributes the load among other hubs mitigating flow congestion (de Camargo, Miranda, Ferreira, Luna, 2009). Hyun and O’Kelly constructed the reliable *p*-hub location problem that focuses on locating *p*-hubs on a network to improve network reliability to deliver interacting flows among its set of origin-destination nodes (Hyun, O’Kelly, 2009). The development of their model relied

on similar concepts for the single and multiple allocation p-hub location problems. A significant contribution to the set of p-hub location problems from their research is the inclusion of reliability of routes in the network. These papers do not necessarily provide relevant model formulations for the CH-47 research scenario of this thesis, but the formulation insight and concept contributions are invaluable to methodology development.

Heuristic Approaches.

Abdinnour-Helm's and Venkataramanan's research into solution techniques for hub-and-spoke similar problems discovered that Morton O'Kelly initially developed a quadratic integer program to solve the p-Hub Median Problem (Abdinnour-Helm and Venkataramanan, 1998). The heuristics developed by O'Kelly to solve his integer program involve complete enumeration of all possible configurations. In his first heuristic, the enumeration process will yield an upper bound on the objective function under the assumption that each spoke node is attached to the nearest hub (O'Kelly, 1987). The second heuristic examines all nodes with respect to its first or second nearest hub. These heuristics offer important insight into the problem at hand, but an additional assumption must be factored in to fix the hubs in-place and address the allocation of nodes to those hubs.

Alternative approaches proposed by Klincewicz involve exchange heuristics which seek to combine multiple metrics to weed out hubs that offer less performance compared to those already found (Klincewicz, 1991). The first approach is a single-exchange heuristic which orders a set of spoke nodes based upon distance and flow traffic and then evaluates those nodes with respect to the its nearest hub(s). The new hub, once

evaluated to a particular new node that offers performance gains, are then swapped for the old hub. The second approach is very similar to the first except it examines a pair of spoke nodes at a time to new hubs; hence, it is called a double-exchange heuristic. Klincewicz's final proposed heuristic is based on clustering. This technique arbitrarily takes a group (cluster) of nodes and then finds the best suited hub based upon the collective metrics of the cluster (Klincewicz, 1991). Again, these heuristics are developed under the scenario that the hub locations are unknown. But clustering techniques may prove useful in solution development to USTRANSCOM's goal of reducing wear on CH-47 blades.

Another author goes on to distinguish between multiple allocation *p*-hub median problems and single allocation ones. Campbell designates those that handle multiple allocations as HMP and those of single allocation as HMP-S (Campbell, 1995). Unless this thesis specifies otherwise, all *p*-hub median problems and their solutions are considered to be of the multiple allocation type. In developing solution heuristics of HMP-S, Campbell utilizes the solution of HMP as a beginning point in obtaining a solution for HMP-S (Campbell, 1995). A greedy-interchange heuristic is used to get a feasible solution to HMP through enumeration and then interchanges hubs that result in a lower transportation cost. Once a solution to HMP is found, all multiple allocations for demand points must be replaced by single allocations (Campbell, 1995). At this point, two heuristics were developed to obtain a solution to HMP-S. The first evaluates maximum flow by linking nodes to the location of the hub in which maximum flow is generated. The second heuristic minimizes transportation cost by evaluating all

combinations of the nodes available. It is important to note that there are no bounding restrictions on the number of demand points.

Aside from *p*-hub median location problems, there is another set of similar hub-and-spoke type problems called uncapacitiated hub location problems, or UHP. The determining distinction of these problems is that the number of hubs is itself a variable. Although the scenario of this research deals with a known number of hubs with known locations, the solution approaches to UHP are still relevant.

In order to solve UHP, Abdinnour generated a new heuristic that blends a Genetic Algorithm (GA) and Tabu Search (TS). Before any further discussion is made on Abdinnour's new heuristic, which is called GATS for "Genetic Algorithm and Tabu Search" (Abdinnour, 1998), the following paragraphs will summarize GA and TS.

GA is derived from the idea of natural selection and genetics to produce a more "evolved" set of solutions given an initial starting solution(s). Each initial solution is evaluated to determine how well it performs. From here a new set of solutions is developed from combining individual solutions from the initial set. This cycle continues until a desired, or "no better," solution is obtained.

In TS, an iterative search procedure is employed such that each iteration moves from one feasible solution to another. Once this move is completed, the algorithm is prohibited to ever go back to a previous feasible solution until a specified number of iterations. This is to prevent cycling. However, each move may not necessarily be an improvement over the previous solution. The idea is to avoid getting trapped in local optimal solutions, i.e. you are forced to test around the feasible region for potentially better solutions.

Abdinnour outlines the new heuristic in five steps. The starting step involves inputting the distance and flow data for the interacting nodes in the network, and initializing the best solution of GATS to a very large number (Abdinnour, 1998). The next step invokes the GA heuristic. Once the best solution is found, it is saved and passed on to the third step, which is the TS phase. TS jumps around the feasible region for a predetermined number of iterations, at which point the best solution is saved and distinguished from the GA solution. The fourth step simply compares solutions between the two sub-heuristic phases. This comparison results in the overall best GATS solution being updated and then steps two through four are repeated for a predefined number of iterations. The final step of the GATS heuristic is the reporting phase.

The last heuristic discussed is a Lagrangean heuristic developed by Elhedhli and Wu. Their approach is intended to solve hub-and-spoke networks formulated as a nonlinear mixed-integer program that explicitly minimizes congestion, capacity acquisition, and transportation costs (Elhedhli and Wu, 2010). The formulated problem is broken into a more solvable subproblem and an NP-Hard nonlinear subproblem. The nonlinear subproblem is first linearized using piecewise functions and then solved to optimality using a cutting plane method (Elhedhli and Wu, 2010). From the subproblem solutions, a heuristic solution can be found to the original problem formulation. This research by Elhedhli and Wu is intended for large-scale networks and takes a hard look at mitigating congestion. The scenario under research in this thesis is not so much concerned with congestion as the underlying network is substantially large utilizing few vehicles, but it does offer useful insight into hub-and-spoke problems modeled as nonlinear programs.

Vehicle-Routing Problems

In the most general sense, the vehicle routing problem can be defined as a set of customers each with a known location and a known requirement for some commodity, which is to be supplied from a set of depots by a set of delivery vehicles of known capacity (Kulkarni and Bhave, 1985). This definition exactly captures the overall goal of this thesis. Under the hub-and-spoke design, our spoke (destination) nodes are the customers that have PAX and cargo requirements from a set of hubs. Currently, the CH-47 is the only delivery vehicle used in this process. The desire is to add the C-130, and potentially other cargo aircraft, to the vehicle set.

Kulkarni and Bhave show that, under certain assumptions, the vehicle-routing problem (VRP) can be reduced to the travelling salesman problem (TSP). Using the TSP as a core model, they show how two formulations for the VRP were developed: single and multiple depot VRPs. In the following formulation, V represents the number of vehicles, P_k is the capacity of vehicle k, T_k is the maximum cost allowed for a route of vehicle k, Q_i is the demand at node I ($Q_N = 0$), and x_{ijk} is a binary variable where it is a one if the pair (i, j) is in the route of vehicle k (Kulkarni and Bhave, 1985). Since the formulation for multiple depot VRPs can be applied to scenarios involving a single depot, only the multiple depot formulation is presented here:

$$\sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{V} c_{ij} x_{ijk}$$

$$subject\ to\ \sum_{i=1}^{N+M} \sum_{k=1}^{V} x_{ijk} = 1\ for\ j = 1,2, \dots, N,$$

$$\sum_{j=1}^{N+M}\sum_{k=1}^{V} x_{ijk} = 1 \; for \; i = 1,2,\dots,N,$$

$$\sum_{i=1}^{N+M} x_{ihk} - \sum_{j=1}^{N+M} x_{hjk} = 0 \; for \; k = 1,2,\dots,V, h = 1,2,\dots,N+M,$$

$$\sum_{i=1}^{N+M} Q_i \sum_{j=1}^{N+M} x_{ijk} \leq P_k \; for \; k = 1,2,\dots,V,$$

$$\sum_{i=1}^{N+M}\sum_{j=1}^{N+M} c_{ij}x_{ijk} \leq T_k \; for \; k = 1,2,\dots,V,$$

$$\sum_{i=N+1}^{N+M}\sum_{j=1}^{N} x_{ijk} \leq 1 \; for \; k = 1,2,\dots,V,$$

$$\sum_{j=N+1}^{N+M}\sum_{i=1}^{N} x_{ijk} \leq 1 \; for \; k = 1,2,\dots,V,$$

$$x_{ijk} = 0 \; or \; 1 \; for \; all \; i,j,k,$$

$$y_i - v_j + (M+N)x_{ijk} \leq N + M - 1 \; for \; 1 \leq i \neq j \leq N \; and \; 1 \leq k \leq V$$

Figure 10 – Kulkarni and Bhave (1985) formulated multiple depot VRP

This formulation assumes that a single vehicle can satisfy a demand node. Additionally, the constraints only permit one vehicle to be assigned to a demand node. Kulkarni and Bhave present a new formulation of the VRP based upon the above model initially formulated by B. L. Golden. In their model, the authors consider the case where vehicle capacity and maximum route cost are identical per vehicle. Kulkarni's and Bhave's new VRP formulation is given below:

$$Minimize\ Z = \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} c_{ij}x_{ij}$$

$$subject\ to \sum_{i=1}^{N+M} x_{ij} = 1\ for\ j = 1,2, \dots, N,$$

$$\sum_{j=1}^{N+M} x_{ij} = 1\ for\ i = 1,2, \dots, N,$$

$$\sum_{i=N+1}^{N+M} \sum_{j=1}^{N+M} x_{ij} = V,$$

$$\sum_{j=N+1}^{N+M} \sum_{i=1}^{N+M} x_{ij} = V,$$

$$x_{ij} = 0\ or\ 1\ for\ all\ i, j,$$

$$y_i - y_j + Lx_{ij} \leq L - 1\ for\ 1 \leq i \neq j \leq N,$$

$$u_i - u_j + Px_{ij} \leq P - Q_i\ for\ 1 \leq i \neq j \leq N,$$

$$v_i - v_j + Tx_{ij} \leq T - c_{ij}\ for\ 1 \leq i \neq j \leq N,$$

Figure 11 – Kulkarni and Bhave (1985) multiple depot VRP model for similar vehicles

Research into the Defense Courier Service distribution network by Baker (1991) explored the effects of the network by reducing the number of depots, shifting the locations, and changing the flight routes by formulating the network as a VRP with an underlying hub-and-spoke structure. The model developed is largely based upon Kulkarni's formulation above along with a formulation by LaPorte (not included in this review). Baker's research led to a model capable of capturing extensive networks that permitted multiple depots and tours, but due to the expansiveness of the problem, it could

not be solved exactly (Baker, 1991). In order to find a solution, a hybrid approach was utilized that combined the minimum spanning forest and the modified Clarke-Wright algorithms.

LaPorte, Nobert, and Taillefer examine three particular variants of VRP/LRPs: capacity constrained VRP, cost constrained VRP, and cost constrained LRP. They contend that producing exact solutions to these problems is still difficult, and subsequently, suggest that other algorithms are more applicable in terms of yielding a feasible solution. In particular, they consider the use of dynamic programming with state-space relaxation in the case of tightly constrained problems, the use of integer linear programming in conjunction with constraint relaxation in the case of loosely constrained symmetrical problems, and branch and bound methods in which every sub-problem is an assignment problem (LaPorte, Nobert, Taillefer, 1988). However, these algorithms can be very time consuming for large problems. For the research purposes of this thesis, these approaches will be potentially suitable as the overall network structure is quite small. Heuristic methods, in contrast, can solve much larger problems and often take into account a greater variety of constraints (LaPorte, Nobert, Taillefer, 1988).

Application to Military Airlift

It is desired to examine several papers that apply past research on VRPs based on (or that has potential to be) a hub-and-spoke network structure. Specifically, past research conducted on the optimization of military airlift assets in routing cargo is discussed. Emphasis is placed on the research’s goals, scenario, variables, and solution technique.

Beginning in 1994, Naval Postgraduate School student Lim Teo-Weng conducted research for a Congressional-sponsored Mobility Requirement Study. Teo-Weng developed a multi-period Strategic Airlift Assets linear program optimization model using the General Algebraic Modeling System (GAMS) (Teo-Weng, 1994). The overall goal of the model is to minimize late deliveries and undelivered cargo. As given parameters, the demand requirements are considered to be known as are the number of available airlift assets and airfields. The desired solution is to find the optimal combination of airlift mission assignments by number and type of aircraft for each unit, routing structure, airlift mission start time and cargo type to carry (Teo-Weng, 1994). The formulated linear program was entered in GAMS which generated a solution to the problem.

A year later, Teo-Weng along with Rosenthal and Morton conducted further research into the strategic airlift assets optimization model developed by Teo-Weng. In their paper, they evaluated the model's performance over scenarios involving the relocation of demand requirements from Ramstein-Riyadh to Dhahran. The formulated model did not change; the goal was to evaluate the model for use in conjunction with other Air Force planning tools. The authors concluded that the model can give a relatively rapid response to questions relating to major mobility issues such as: 1) Are the given aircraft and airfield assets adequate for the deployment scenario? 2) What are the impacts of shortfalls in airlift capability? 3) Where are the system bottlenecks and when will they become noticeable? (Morton, Rosenthal, Teo-Weng, 1995).

In 1997, RAND briefed the outcome of a study requested two years earlier to evaluate how the C-17 could be used as an in-theater airlifter. Specifically, the study was

to determine the appropriate mix of C-17s and civil-derived aircraft in the U.S. military airlift fleet (Killingsworth, Melody, 1997). The model had thousands of constraints and variables. Conceptually, the formulated model is fundamentally a VRP altered to the specific scenario and given parameters. As in the research conducted by Teo-Weng, the objective of the model is to minimize late deliveries and non-deliveries through a weighted penalty coefficient. The model was solved using the GAMS software.

Cox developed a hub-and-spoke combined location-routing mixed integer programming model in 1998 as an alternative to, at the time, the U.S. Air Force's current directly delivery methodology. In the underlying network structure, he looks at three types of nodes: those that act as hubs, transshipment nodes, and destination nodes (Cox, 1998). His model incorporated many aspects from some of the authors previously discussed in this literature review, particularly the works of Kulkarni and Bhave, and Baker. The objective function minimizes lateness and non-deliveries, and utilizes the following variables:

- X^h_{ij} is a binary variable equal to 1 if aircraft with tail number *h* flies the arc from *i* to *j*
- X_{ij} is a binary variable equal to 1 if demand node *j* is supplied by a plane based at depot *i*
- Z_{0j} is a real-valued variable, equal to the amount of cargo delivered from all supply nodes to transshipment (depot) node *j*
- Y_i is a binary variable equal to 1 if any aircraft are assigned to depot *i*
- T^h_j is the time that aircraft *h* spends at node *j* (Cox, 1998)

The model was solved using CPLEX 3.0 which generated a solution in less than a minute. Cox concluded that the hub-and-spoke model developed was more advantageous to the Air Force versus the current method of conducting business.

On a larger scale network, Skipper (2002) analyzed the distribution network of U.S. European Command by formulating a multiple objective linear programming model. The goal was to find an optimal set of hubs that would reduce transportation costs and time required for cargo shipments. Constructing the model required identifying the current network including the supply node, the current hub location, and the demand nodes. The variable used in the development of the model was X_{ij} which represents the number of missions from node i to destination j (Skipper, 2002). Several different scenarios where different locations acted as a hub for the network were modeled. Excel was used as the solution tool.

That same year, INFORMS published an article by Baker, Morton, Rosenthal, and Williams (2002) that summarized a massive combined effort on the part of the Naval Postgraduate School and the RAND Corporation that developed the NPS/RAND Mobility Optimizer (NRMO) model. To date, this is one of the most all-encompassing optimization models developed. Overall, the model utilizes a similar set of general categories of variables such as which aircraft is delivering to a particular node and what aircraft is being employed for making deliveries. However, NRMO breaks these variables down into much more detailed components resulting in a significantly higher number of variables. GAMS in conjunction with the parallel CPLEX 6.0 barrier algorithm was used to generate a solution (Baker, Morton, Rosenthal, Williams, 2002).

Lockhart (2005) examined whether the C-27J aircraft would be a viable solution to help increase intratheater airlift efficiency in his research paper in 2005. The particular methodology utilized in this research represents a major departure from the solution techniques presented earlier. In solving the problem, the researcher conducted an analysis of quantitative data to evaluate potential efficiency improvements if the C-27J is added to U.S. airlift capabilities. Specifically, a load factor calculation was developed to determine if it would be more efficient for the C-27J to carry a load that would otherwise be carried on a minimally loaded C-130 (Lockhart, 2005). Lockhart proposes an alternate methodology which could be used to evaluate the potential replacement of several CH-47s with a couple of C-130s.

A more recent paper by Rivera (2009) conducted research to determine the right mix of airlift assets that would be most suitable for moving personnel and cargo along the Last Tactical Mile. Taking a more data analytic approach, Rivera used data from current operations in Afghanistan as a baseline to evaluate the effectiveness of potentially employing the use of the C-130J-30, C-27J, and CH-47 aircraft along specified routes (Rivera, 2009). His paper suggests that some mathematical models were formulated to help form a solution, but these are never presented. However, the bulk of his solution technique consisted of injecting the baseline data into the Air Tasking Efficiency Model and then evaluating each of the above specified aircraft.

Summary

The problem of reducing wear on the CH-47 blades through the incorporation of additional cargo aircraft, particularly the C-130, involves many aspects of research on hub-and-spoke VRPs already accomplished. The intent of this literature review is to

demonstrate what mathematical formulations are currently available and proven to work. Of the papers reviewed, it is evident that none can provide a direct solution to this problem. Collectively, these past research endeavors provide a foundation on which to formulate a mathematical integer linear program unique to CH-47 scenario. Although there are many problem formulations for hub-and-spoke networks, the real goal is to determine if some CH-47s can be replaced by C-130s. This leads to the VRP as the fundamental problem that needs to be solved over a hub-and-spoke network. The last section examined how this combined problem has been applied to military airlift mobility problems in the past along with practical solution techniques.

III. Methodology

Introduction

It is desired to reduce the number of CH-47 hours flown currently employed in the Iraq and Afghanistan theater of operations. In order to accomplish this, several methods were fathomed in relation to a mathematical programming approach. An initial proposed method was to begin with a generalized vehicle-routing problem (VRP) and then add or modify constraints specific to USTRANSCOM's scenario. However, VRPs in the most general sense use a single vehicle, or, if multiple vehicles are utilized, lack the detail on an individual vehicle level to produce the information required for this research.

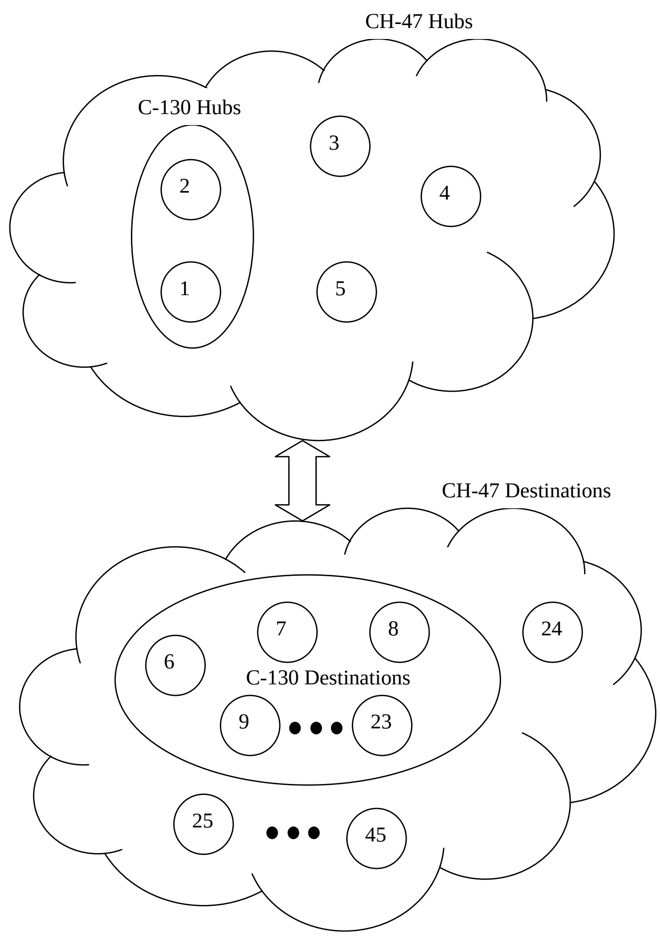
Since this research focuses on a multiple aircraft-type scenario where time-tracking on each vehicle is necessary, a more detailed model was required in which additional aircraft-types could be added and easily tracked. This leads to the proposed methodology for this research. The following sections detail a mathematical programming model with an underlying hub-and-spoke network that follows a generalized network flow problem. By 'generalized,' it is meant that an entity (aircraft) must travel out of a source node (hub), traverse to a destination or series of destinations, and then proceed to a sink node (same hub).

Case Study/Network Mapping

The real-world operational network involves forty-five U.S. bases in the Iraq and Afghanistan theaters. Due to the sensitivity of the data, these bases will not be named, but a node-base relationship table can be established in any fashion so long as those

nodes acting as hubs are at the beginning of the sequence. These bases collectively make up the entire network, or node set. Five of the bases form a subset of nodes that act as the hubs for the network. The remaining forty bases are the destination nodes, or the nodes that have both passenger (PAX) and cargo requirements. Additionally, these destination nodes can be formed into subsets for each aircraft-type as there are base restrictions on whether a destination node can accommodate a particular aircraft type. For this scenario, the CH-47 has full access to every hub and node in the network. The C-130, on the other hand, is only capable of flying into eighteen nodes from two hubs. The network flow diagram in figure 1 captures these scenarios:

Figure 12 – General Network Flow



The top cloud represents the hubs of the network where a number of available CH-47 aircraft are stored. Only two CH-47 hubs can accommodate the C-130 aircraft. The bottom cloud consists of nodes that comprise the serviceable destinations, each with PAX and cargo requirements (accounted as a single total weight) for any given day.

Several assumptions apply to this network. The first is that there are no PAX or cargo requirements between the five hubs. All aircraft originate at the hubs and must travel to the destination nodes to pick up and transport passengers and cargo back to the hub. Secondly, aircraft do not travel between hubs. That is, all aircraft simply perform sorties from their respective hub to a single destination or multiple destinations and back. Multiple aircraft can service a single destination. Lastly, there must be enough aircraft available to completely transport all PAX and cargo.

Integer Program Development

In accordance with the prescribed methodology, the variables of interest are the number of aircraft formations required to service all the PAX and cargo requirements for a destination node for each type of aircraft. By aircraft formations, it is meant to capture any number of aircraft that fly together during a sortie. For example, it is required that CH-47s always fly in pairs. Hence, a CH-47 formation in this model is a two-ship formation. However, any number of aircraft flying together can be specified in order to model multi-ship formations. The variables account for both CH-47s and C-130s. In order to establish a baseline scenario in which only CH-47s are utilized, the C-130 variables are 'zeroed' out by forcing the number of C-130 aircraft available to zero.

$$SORTIE_{T,H,N,I,J}$$

$$= \begin{cases} 1\ if\ arc(I,J) is\ traversed\ by\ vehicle\ type\ T, assigned\ to\ hub\ H, number\ N \\ 0, otherwise \end{cases}$$

$$where \begin{cases} T = 1\ (CH47)\ or\ 2\ (C130) \\ H = 1, 2, \dots, 5 \\ N = 1, 2, \dots, 7 \\ I = 1, 2, \dots, 15 \\ J = 1, 2, \dots, 15 \end{cases}$$

$$additionally\ I\ \neq J$$

NUM_REQ_T = number of aircraft type T formations needed to transport all cargo requirements from every destination (*)

Additionally, must be integer and greater than or equal to zero

*Note: NUM_REQ_{CH47} and NUM_REQ_{C130} are utilized in unison so each variable is dependent upon the number of other vehicle type used.

$$V\_USED_{T,H,N} = \begin{cases} 1\ if\ vehicle\ type\ T, assigned\ to\ hub\ H, number\ N\ is\ utilized \\ 0\ otherwise \\ H = 1, 2, \dots, 5\ if\ t = 1 \\ H = 1, 2\ if\ t = 2 \end{cases}$$

$$CT_{T,H,N,J} = \begin{cases} amount\ of\ cargo\ taken\ from\ destination\ J\ by\ vehicle\ T, \\ assigned\ to\ hub\ H, number\ N \end{cases}$$

$$where \begin{cases} \forall\ T, H, N, J \\ must\ be\ \geq 0 \end{cases}$$

$$PT_{T,H,N,J} = \begin{cases} amount\ of\ PAX\ taken\ from\ destination\ J\ by\ vehicle\ T, \\ assigned\ to\ hub\ H, number\ N \end{cases}$$

$$where \begin{cases} \forall\ T, H, N, J \\ must\ be\ integer\ and \geq 0 \end{cases}$$

A full scenario will require 291,735 variables and 366,415 constraints. This assumes seven aircraft slots are available at each of the five hubs for each aircraft type, and at each of the 45 destinations. Again, these numbers are dependent on number of nodes, hubs, aircraft types, and number of available aircraft.

The goal for this mixed integer linear program (MILP) is to minimize the total costs for each aircraft type traveling along each hub-destination pair. Using data from U.S. Transportation Command (USTRANSCOM) on flight duration between hub and nodes and DoD estimates on operating costs per hour for each aircraft type, a general cost can be determined based on aircraft type and hub-node pair being traveled. Additionally, since the CH-47s fly in pairs, the costs associated a CH-47 sortie must be doubled. The following equation defines the objective function for this ILP:

$$Min\ w = \sum_{T} \sum_{H} \sum_{N} \sum_{I} \sum_{J} (COST_T * NUM\_FORMATION_T * TIME_{T,I,J} * SORTIE_{T,H,N,I,J})$$

$$where\ I \neq J$$

The limitations that occur when aircraft type 2 (C-130) is utilized are due to restrictions on hubs and destinations that can accommodate the C-130 aircraft. In this scenario, C-

130s can only fly out of hubs one and two, and can only service the first eighteen nodes (nodes six through twenty-three). The $COST_T$ vector contains DoD operating costs per flight hour of the CH-47 and C-130 in 2007. $NUM\_FORMATION_T$ is a vector that specifies the number of aircraft that travel together by type T. The $TIME_{T,I,J}$ parameter is an n-by-n matrix where $n = J$ or I and contains the flight time (in hours) between all arc pairs (I, J). Within the matrix, if $I = J$, then an arbitrarily high number is used to ensure to no destination is traveled to itself. However, these values are place holders as variables that contain $I = J$ pairs are not created. Cost and flight time data is provided by USTRANSCOM.

The constraints for this problem have been broken into five primary areas. The first area determines the number of aircraft formations required to transport all PAX and cargo from all the destinations. This constraint requires that there is sufficient available aircraft to do so. For the PAX and cargo requirements (second area), only the total weight of both PAX and cargo is considered and not the volume. A key assumption here is that a “full aircraft capacity” is reached when fifty-five percent of the aircraft has been filled in terms of weight (lbs). The third set restricts aircraft flow throughout the network. This set maintains the network flow relationship between hub, destination, and aircraft type. For example, a CH-47 has access to every base that a C-130 does, but the C-130s do not have access to every CH-47 base. The fourth area enforces hour limitations on each aircraft type. Lastly, the fifth set places integer restrictions on the variables, some of which are binary.

Aircraft Required.

Three constraints determine how many of each aircraft type are needed and establish which of the available aircraft will fly sorties. The total weight capacities for each aircraft type are used as parameters here to determine the number of aircraft required. However, since this model does not account for the volume space of both cargo and aircraft, each aircraft type's capacity is multiplied by an adjustment factor in order to account for scenarios where an aircraft's cargo volume is fully used, but is not at weight capacity. Based on historical data from USTRANSCOM, this adjustment factor is fifty-five percent.

Constraint 1a – Ensures sufficient vehicle capacity is available:

$$\sum_{T} NUM\_FORMATION_T * WT\_CAP\_ADJ_T * WT\_CAP_T * NUM\_REQ_T \geq \sum_{J} CARGO\_REQ_J * WT\_PER\_PAX * PAX\_REQ_T$$

Where:

$WT\_CAP\_ADJ_T$ contains the adjustment factor for each vehicle type,

WT_CAP_T contains the weight capacity by vehicle type,

$CARGO\_REQ_J$ is a vector specifying cargo requirements at node J,

WT_PER_PAX is a planning parameter indicating weight for each passenger,

and PAX_REQ_T is a vector specifying PAX requirements at node J

Constraint 1b – Ensures total vehicle fleet capacity does not exceed a factor for two:

$$\sum_{T} NUM\_FORMATION_T * WT\_CAP\_ADJ_T * WT\_CAP_T * NUM\_REQ_T$$
$$\leq 2 * \sum_{J} (CARGO\_REQ_J * WT\_PER\_PAX * PAX\_REQ_T)$$

Constraint 2a – Initializes vehicles to be used based on need:

$$\sum_{H} \sum_{N} V\_USED_{T,H,N} \geq NUM\_REQ_T \quad \forall\, T$$

Constraints 2b – Vehicles used cannot exceed vehicles available at a hub:

$$\sum_{N} V\_USED_{T,H,N} \leq AVAIL_{T,H} \quad \forall\, T, H$$

$where\ AVAIL_{T,H}\ is\ a\ vector\ specifying\ number\ of\ vehicles\ by\ type, assigned\ to\ each\ hub$

PAX and Cargo Constraints.

PAX and cargo are both accounted for as a total weight requirement at each destination that needs to be transported to a hub. There is no restriction on which hub that the cargo must be flown to nor is the portion of weight distinguished between PAX and cargo. The following constraints restrict an aircraft from overloading as it makes stops at various destinations as well as enforces that the cargo taken on at any single destination does not exceed the allotted aircraft weight capacity. This latter constraint is likely to be redundant in most cases.

Constraints 3 – Vehicle cargo/PAX weight capacity restriction:

$$\sum_{J}(CT_{T,H,N,J} + WT_{PER_{PAX}} * PT_{T,H,N,J})$$
$$\leq NUM\_FORMATION_{T} * WT\_CAP\_ADJ_{T} * WT\_CAP_{T}$$
$$* V\_USED_{T,H,N} \ \forall \ T, H, N$$

Constraint 4 – Vehicle allocated weight capacity for cargo:

$$\sum_{J} CT_{T,H,N,J} \leq NUM\_FORMATION_{T} * MAX\_CARGO\_WT_{T} * V\_USED_{T,H,N} \ \ \forall \ \ T, H, N$$

where $MAX\_CARGO\_WT_T$ specifies maximum amount of a vehicle's weight capacity allotted to cargo; must be less than or equal to WT_CAP_T

Constraints 5 – Vehicle allocated seating capacity for PAX:

$$\sum_{J} PT_{T,H,N,J} \leq NUM\_FORMATION_{T} * MAX\_PAX_{T} * V\_USED_{T,H,N} \ \ \forall \ \ T, H, N$$

where MAX_PAX_T specifies the maximum number of passengers able to be carried on vehicle type T; must be integer

Constraint 6 – Ensures all cargo is transported:

$$\sum_{T}\sum_{H}\sum_{N} CT_{T,H,N,J} = CARGO\_REQ_{J} \ \ \forall \ \ J$$

Constraint 7 – Ensures all PAX is transported:

$$\sum_{T}\sum_{H}\sum_{N} PT_{T,H,N,J} = PAX\_REQ_{J} \ \ \forall \ \ J$$

Constraint 8 – Ensures picked up cargo does not exceed vehicle cargo capacity:

$$CT_{T,H,N,J} \leq NUM\_FORMATION_{T} * WT\_CAP\_ADJ_{T} * WT\_CAP_{T}$$
$$* \sum_{I} SORTIE_{T,H,N,I,J} * SWITCH_{T,I,J} \ \ \forall \ T, H, N, J$$

where $SWITCH_{T,I,J}$ is a user-defined *n*-by-*n* binary (0-1) matrix that acts as toggle switches to control where a vehicle may travel, and $I \neq J$

Constraint 9 – Ensures picked up PAX does not exceed vehicle PAX capacity:

$$PT_{T,H,N,J} \leq NUM\_FORMATION_T * MAX\_PAX_T * \sum_{I} (SORTIE_{T,H,N,I,J} * SWITCH_{T,I,J}) \; \forall \; T,H,N,J$$

where I ≠ J

Network Flow Constraints.

Aircraft movement throughout the network is modeled utilizing a general flow model involving distinct source and sink nodes. In this research, a hub is considered to be both. Since the network contains undirected arcs, flows out of a node are considered to be positive flows whereas those into a node are negative. This provides a simple flow structure that ensures aircraft leave (if needed) their assigned hub and return to it. In order to generate movement, the hub from a source node perspective is given a flow of a single positive unit and, as a sink node, is given a flow of a single negative unit. That is, a hub node has one unit (an aircraft) that must be flowed to itself. Intermediate nodes destinations have a requirement of zero flow, which forces an aircraft to stop at a destination only if there is a cargo requirement. However, an easily solvable problem arises when a node is utilized as both source and sink, and the flow requirement is zero (instances when an aircraft is not used). If both source and sink network flows require zero units to be moved, this permits the intermediate network flows to potentially transit an "imaginary aircraft" between an out-flowing and in-flowing arc pair, which will take on any cargo required at those nodes forming the arc. To break this occurrence, the sum

of all arc pairs (I, J) and (I, I) when I and J are greater than the largest number of hubs must be less than or equal to a unit being flowed, if any. This not only prohibits these occurrences, it also acts as a redundant constraint in preventing aircraft from returning to a previously visited node.

Constraints 10a – Source node (hub):

$$\sum_J (SORTIE_{T,H,N,H,J} * SWITCH_{T,H,J}) = V\_USED_{T,H,N} \quad \forall\, T, H, N \ and\, J \neq H$$

Constraints 10b – Destination restrictions:

$$\sum_J (-SORTIE_{T,H,N,I,J} * SWITCH_{T,I,J} + SORTIE_{T,H,N,J,I} * SWITCH_{T,J,I})$$

$$= 0 \quad \forall\, T, H, N, J$$

$$where \begin{cases} I \neq J \\ J \neq \mathrm{H} \end{cases}$$

Constraints 10c – Sink node (hub):

$$\sum_J (-SORTIE_{T,H,N,J,H} * SWITCH_{T,J,H}) = -V\_USED_{T,H,N} \quad \forall\, T, H, N \ and\, J \neq H$$

Constraints 10d – Ensures destinations are not used as hubs:

$$SORTIE_{T,H,N,I,J} * SWITCH_{T,I,J} + SORTIE_{T,H,N,J,I} * SWITCH_{T,J,I}$$

$$\leq V\_USED_{T,H,N} \ \forall\, T, H, N, I, J$$

$$where \begin{cases} I \neq J \\ I \neq H \\ J \neq H \end{cases}$$

Constraints 10e – Sub-tour elimination:

$$\sum_{I}(SORTIE_{T,H,N,I,J} * SWITCH_{T,I,J} + SORTIE_{T,H,N,J,I} * SWITCH_{T,J,I})$$

$$\leq 3 - SORTIE_{T,H,N,H,K} * SWITCH_{T,H,K} - SORTIE_{T,H,N,K,H}$$

$$* SWITCH_{T,K,H} \quad \forall J, K$$

$$where \begin{cases} K \neq J \\ J \neq H \\ I \neq H \\ I \neq J \end{cases}$$

In scenarios involving multiple vehicle types, the vehicle type which utilizes the largest number of hubs establishes the vector length for the $AVAIL_T$ vector. Those vehicles that do not utilize a certain hub are zeroed out such that they cannot be based out of that particular hub. Additionally, there must be a restriction on sorties when *I* equal *J*:

Constraint 11 – Ensures no node self-traveling:

$$SORTIE_{T,H,N,I,J} = 0 \quad \forall T, H, N, I, J \text{ and when } I = J$$

Aircraft Hour Limitations.

Both the CH-47 and the C-130 have total operating hour restrictions. A six hour limitation is placed on the CH-47 while the C-130 can fly up to twelve hours.

Constraints 12 – Vehicle travel duration restrictions:

$$\sum_{I}\sum_{J}(TIME_{T,I,J} \times SORTIE_{T,H,N,I,J}) \leq DURATION\_LIMIT_T \quad \forall T, H, N, I, J \text{ and } I \neq J$$

where $DURATION\_LIMIT_T$ is a vector specifying the time (in hours) limitation for which a vehicle type can operate continuously

Variable Constraints.

The sortie and aircraft used variables must be binary such that they indicate whether a particular arc (I, J) was flown and whether a particular aircraft of those available was flown. The variables used for determining how many aircraft are required are general integers. Cargo is not restricted to be integer, and inherently assumes that the largest piece of cargo may be loaded onto either vehicle types.

Constraint 13a – Required number of vehicles:

$$NUM\_REQ_T \geq 0 \; and \; integer \quad \forall \, T$$

Constraint 13b – Number of PAX picked up:

$$PT_{T,H,N,J} \geq 0 \; and \; integer \quad \forall \, T, H, N, J$$

Constraint 14 – Specific vehicle utilization:

$$V\_USED_{T,H,N} \in [0,1] \; and \; integer \quad \forall \, T, H, N$$

Constraint 15 – Travel between node I and node J:

$$SORTIE_{T,H,N,I,J} \in [0,1] \; and \; integer \quad \forall \, T, H, N, I, J$$

LINGO-based Model Development

The formulated model in the preceding section was solved utilizing LINGO ® version 12.0.1.10 dated 18 January 2010. This formulation consisted of 16,882 integer variables defining the number of aircraft by type and number that originate from a particular hub and travel to a particular destination J from destination I. This also includes variables for the cargo taken, aircraft number referencing, and the number of required aircraft. There are 44,715 constraints total that enforce network flow, PAX and weight capacities, and flight time limitations. The model was executed on a HP computer system, model DX5150MT with an Athlon 64 X2 4400+ processor and halted after ten

minutes of run time. Initial modeling trials were conducted up to sixteen hours resulting in no or minuscule changes in the objective function. In general, the model reaches a "stable" feasible solution after a few minutes. Ten minutes of runtime is selected to ensure an adequate amount of time has passed and no changes in the objective function occurred. On a side note, the objective function changes observed while permitting the model to run for consecutive hours is on the order of a couple $1000. In relation to a objective function that generally totals over $100,000, the time required to obtain such small gains is deemed impractical.

Through the objective function minimizing cost to service the destination bases, the solution, if one exists, table displays the overall cost for meeting all destination requirements and integer values for each variable. A strict interpretation of the sortie variables should read as 'a type t aircraft, number n assigned to hub h flew a sortie from destination i to destination j.' For practical purposes, it may not be necessary to send multiple aircraft to a particular destination even though a model solution may indicate multiple aircraft are needed. Say for example, two aircraft are needed to service a destination according to this model, but practically only one aircraft is needed to perform two round-trips. In either case, this does not affect the overall cost. Ultimately, it is the decision maker's interpretation whether multiple aircraft are absolutely required or if fewer aircraft are required making multiple trips.

In developing this model, it is intended to capture all possible destination requirements on any given day. However, the Iraq and Afghanistan data on which this model is based demonstrates that on an average day, only five to seven destinations have requirements. This means that many of the constraints can be zeroed-out, or 'switched

off,’ greatly reducing the overall model and runtime (this can be a significant issue when all destinations have requirements). Should obtaining a solution in a short amount of time be required, the model does generate a ‘good enough’ (feasible) solution in a matter of minutes or less. Subsequently, this means that for a given day, all constraints will have to be altered for that day’s destination requirements.

With respect to updating the model, some background on linear program will be required should it be desired to include additional aircraft types and/or bases that open or close links. However, for updating the daily PAX and cargo weight requirements, a simple number change in the corresponding data table is sufficient. This also applies to updating the number of available aircraft stationed. A cautionary note for updating any daily requirements and aircraft availability is to ensure one knows the relationship between hub/destination number assignment and the actual base name.

A last essential note in executing this model is that it is mandatory to have all daily requirements satisfied. This means if there are less aircraft available than needed to meet all requirements, the model will not be able to generate a solution (i.e. no feasible solution will be found). However, as noted earlier concerning the practical use of aircraft making roundtrips, this model can be “short-sighted” in terms of using few aircraft to service all destinations. Still, this scenario, though a possibility, is not likely to occur given the current (as of July 2010) number of aircraft available at the nodes and the low number of destinations that generally have requirements on a given day.

Summary

The integer linear program formulated and executed in this research does have both its merits and limitations. The lack of connections between hubs significantly

reduces underlying network structure and ultimately loses its similarity with traditional hub-and-spoke problems. This significantly generalizes the overall model and permits relatively simple manipulation through adjusting aircraft availability numbers. Using this manipulation, an experimental design can be developed to test various scenarios containing different aircraft types and the increase/decrease of aircraft numbers in order to evaluate the savings in CH-47 blade hours as well as the overall cost. However, utilizing aircraft with higher operating costs, such as the C-130, may prove to save CH-47 blade hours at more monetary expense.

Revisiting the research objectives from Chapter 1 on page 2, this model can accomplish all of these. In regards to the first objective of saving CH-47 blade hours, every aircraft type that services a destination results in airframe savings for the CH-47. The model formulated in this research generates data that can be used to calculate those savings. The second objective seeks to determine the amount of C-130 efforts to generate the CH-47 blade hour savings. Both the number of C-130s and the associated cost of using those C-130s yields an effective determination of effort required to obtain the desired savings. Lastly, the model itself is a realization of achieving the third objective of this research. With this model, an optimized mix of CH-47 and C-130 aircraft can be found while satisfying all anticipated PAX and cargo requirements in the Iraq and Afghanistan theater. Additionally, the model lends itself to easy experimentation that permits forced reduction of CH-47 aircraft in order to generate blade hour savings.

IV. Results and Analysis

Introduction

The model analysis provided in the following section is based on results for a smaller-scale scenario. This was necessitated in order to perform multiple runs of the model and obtain a feasible solution in a reasonable amount of time. Full-up scale scenarios are conducted as demonstrations of the model's ability to find a useable solution, but they require a much longer runtime. A specified scenario where only CH-47s are employed serves as a baseline for analysis.

Destination node cargo requirements serve as the primary drivers in flowing vehicles throughout the modeled network. Model results are based on achieving 100% cargo movement to a hub in the most cost effective manner utilizing the air assets available at each hub. An important note needs to be stated in relation to cargo. From the model's perspective, the number of PAX in terms of weight (lbs) and storage cargo weight are indistinguishable. Cargo requirements at a destination are in reference to the collective amount of PAX and storage cargo weight required to be moved. Moreover, where cargo is moved is dependent on which aircraft picks up the cargo.

Scenario Development

The underlying model's network is designed to permit complete access from one node to any other node in the network aside from those nodes that act as hubs. There is no limit prohibiting a user from entering as many nodes as desired so long as the computer hardware contains sufficient memory. This open-ended style is selected to permit this model to be employed in a vast number of other scenarios if so desired. For

this particular research scenario, there are forty-five bases considered in the Iraq theater. Five of these bases act as major hubs for the aircraft of interest, the CH-47 and C-130. With the forty-five bases alone, 2,025 arcs are generated that translate into variables that need to be tracked. However, we also need to track the different aircraft types, the number of aircraft for each type available, and to which hub the aircraft is assigned. This leads to a combinatorial variable explosion such that to exhaust every possibility in search of the most cost efficient combination would be impractical in terms of computer runtime.

The full-scale scenario mentioned above for this research results in just over 290,000 variables. However, in review of CH-47 flight logs in the Iraq theater, it is found that, on any given day, only six to seven of the destination bases had cargo requiring movement. To execute the full-scale scenario with those destinations that had no requirements zeroed out would still be taxing in terms of computer runtime. By employing a small-scale scenario with fewer destination nodes, a feasible solution can be found much quicker while still providing practical results under a realistic setting.

Scenario for Model Analysis.

It is decided to demonstrate the potential savings in CH-47s blade hours by utilizing a realistic scenario set-up with data emulating CH-47 and C-130 flight time (in hours) between all node combinations. This scenario consists of fifteen nodes total, the first five which serve as major hubs for the area. CH-47s are stationed at every hub where as C-130s are only stationed at the first two hubs. Additionally, CH-47s have access to every destination node in the network (i.e. nodes six through fifteen). The C-130s are only capable of servicing nodes six through ten due to airfield restrictions at the

remaining destinations. Random cargo requirements in terms of thousands of pounds were generated for each destination. Actual aircraft weight capacities are implemented, but reduced by a 55% scaling factor in order to reflect historical aircraft loading data that indicates, on average, that only 55% percent of an aircraft's total weight capacity is utilized due to an aircraft's volume limitations being reached. Each aircraft's cost per flying hour is based on DoD figures from 2007, data provided by USTRANSCOM. Lastly, the current (as of 2009) number of CH-47s available at each of the five major hubs in the Iraq theater are also implemented here.

The general scenario established above is evaluated in two ways. The first method consisted of incrementally permitting C-130 aircraft to be available for use at particular hubs, which in effect allows the model to determine if a C-130 should be utilized if it reduces the cost of cargo delivery. The second method forces a number of C-130 aircraft to be utilized, but permits the model to determine which non-restricted hub to assign the aircraft such that costs can be minimized. Based on the generated cargo requirements, the aircraft weight capacities, and node access restrictions, the most C-130s that can be employed is three. For those destinations which cannot be serviced by C-130s, it will require a minimum of eighteen CH-47s (or nine formations of two CH-47s).

Results and Analysis

Scenario.

For the established scenario settings above, the two scenarios were executed several times each as the number of C-130 aircraft is incremented. The first scenario, which references the option of utilizing any C-130 aircraft made available, evaluates each increment of C-130 at each non-restricted hub possibility. In the second scenario, the

number of C-130s forced to be utilized is simply evaluated and compared to the relatable instance in scenario one. An instance where only CH-47s are flown serves as the benchmark with which to compare the two scenarios. This benchmark simulates the current CH-47s operations in Iraq and Afghanistan theaters such that no C-130s are being flown to augment the CH-47 workload.

For the benchmark run, no C-130s were available at hubs one or two. The model is executed for ten minutes and the resulting feasible solution is evaluated. It is reported that fifteen CH-47 formations (thirty individual aircraft) were required to transit all cargo requirements from the destinations to a hub. A simple calculation of summing the total cargo requirement weights and dividing by the adjusted weight capacity for a CH-47 verifies that this is indeed the minimum number of aircraft required to move all cargo for this scenario. Totaling all the sorties flown, the solution generated required 61.88 flying hours, resulting in an objective function cost of $168,499. These figures are in reference to CH-47 total flying hours and cost for a day.

With a captured CH-47 workload baseline, the first scenario is executed utilizing all possible C-130 increments (up to three) and hub assignments. The results of these trials are shown below in Table 1 and Table 2 with reduction values in relation to the baseline:

Table 1 - Scenario 1 (Optional C-130s) Reference		
Scenario	# of C-130s at Hub 1	# of C-130s at Hub 2
S1-1	1	0
S1-2	0	1
S1-3	2	0
S1-4	0	2
S1-5	1	1
S1-6	3	0
S1-7	0	3
S1-8	2	1
S1-9	1	2

Table 2 - Scenario 1 (Optional C-130s) Results										
Scenario	Flight Time		Costs			CH-47 Time Reduction		Cost Reduction		Vehicle Eff
	CH-47	C-130	CH-47	C-130	Total	Time	%	Cost	%	
S1-1	49.68	1.15	$135,278.60	$4,197.50	$139,476.10	12.2	19.72%	$29,023.10	17.22%	1.05
S1-2	49.32	1.96	$134,298.40	$7,154.00	$141,452.40	12.56	20.30%	$27,046.80	16.05%	1.05
S1-3	40.62	2.76	$110,608.30	$10,074.00	$120,682.30	21.26	34.36%	$47,816.90	28.38%	1.03
S1-4	41.38	4.95	$112,677.70	$18,067.50	$130,745.20	20.5	33.13%	$37,754.00	22.41%	1.03
S1-5	43.22	3.28	$117,688.10	$11,972.00	$129,660.10	18.66	30.16%	$38,839.10	23.05%	1.03
S1-6	37.2	3.13	$101,295.60	$11,424.50	$112,720.10	24.68	39.88%	$55,779.10	33.10%	1.14
S1-7	40.02	4.22	$108,974.50	$15,403.00	$124,377.50	21.86	35.33%	$44,121.70	26.19%	1.2
S1-8	38.5	3.08	$104,835.50	$11,242.00	$116,077.50	23.38	37.78%	$52,421.70	31.11%	1.14
S1-9	40.46	3.28	$110,172.60	$11,972.00	$122,144.60	21.42	34.62%	$46,354.60	27.51%	1.21

Depending on hub assignment, allotting one C-130 to augment the CH-47's daily workload reduced the workload up to approximately twenty percent, or saved around twelve hours per day of wear on the CH-47's airframe and blades. Additionally, the overall cost for transiting all cargo requirements is reduced by sixteen percent. Trials six through nine, which utilize the maximum number of C-130s that can be made available, demonstrate savings on CH-47 blade hours that average around thirty-six percent. Overall costs were reduced by approximately twenty-nine percent.

Scenario two, which required the designated number of C-130 aircraft to conduct sorties, generated similar results to scenario one in relation to the same increment in C-130 aircraft. Tables 3 and 4 contain the results of scenario two trials:

Table 3 - Scenario 2 (Forced C-130s) Reference	
Scenario	# of C-130s required to fly
S2-1	1
S2-2	2
S2-3	3
*S2-4	4
*S2-5	5
*S2-6	6

Table 4 - Scenario 2 (Forced C-130s) Results										
Scenario	Flight Time		Costs			CH-47 Time Reduction		Cost Reduction		Vehicle Eff
	CH-47	C-130	CH-47	C-130	Total	Time	%	Cost	%	
S2-1	50.54	1.56	$137,620.40	$5,694.00	$143,314.40	11.34	18.33%	$25,184.80	14.95%	1.05
S2-2	43.18	3.35	$117,579.10	$12,227.50	$129,806.60	18.7	30.22%	$38,692.60	22.96%	1.03
S2-3	38.26	3.43	$104,182.00	$12,519.50	$116,701.50	23.62	38.17%	$51,797.70	30.74%	1.14
*S2-4	37.38	3.67	$101,785.70	$13,395.50	$115,181.20	24.5	39.59%	$53,318.00	31.64%	1.32
*S2-5	37.34	4.05	$101,676.80	$14,782.50	$116,459.30	24.54	39.66%	$52,039.90	30.88%	1.5
*S2-6	37.38	4.26	$101,785.70	$15,549.00	$117,334.70	24.5	39.59%	$51,164.50	30.36%	1.67

Similar to scenario one, the model is allowed to run these trials for ten minutes and the best feasible solution to the model at the end of those ten minutes is reported in Table 4. Comparing the scenario-trial references, S2-1 in relation to S1-1 and S1-2, it is evident that both scenarios yielded similar results. Additionally, it should be observed that the model selected the forced C-130 to be assigned at Hub one to be more cost efficient. Similar observations can be made to S2-2 and S2-3 in regards to the same number of C-130s made available in scenario one. The asterisked scenario two trials, four through six, were conducted as additional model experiments. From a high level perspective, the use of subsequent C-130 aircraft outside of the theoretical maximum number of these aircraft reflected relatively no difference in the overall CH-47 hour savings and overall daily operating cost. Looking at the results more in-depth, the excess

C-130s either spread the cargo loads between them or flew a sortie along the shortest route and returned to its hub without taking on cargo. Figures 13 and 14 summarize the CH-47 flight time savings and cost reductions:

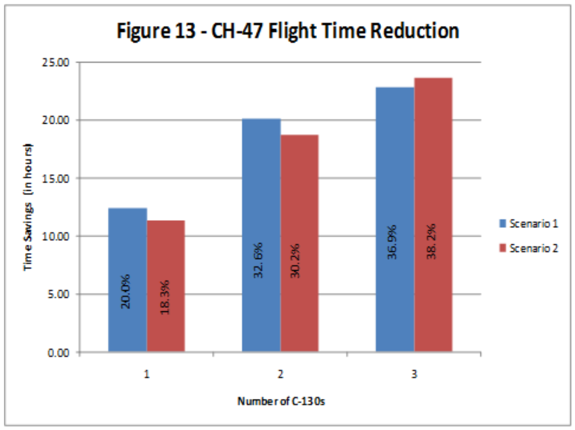


Figure 13 - CH-47 Flight Time Reduction

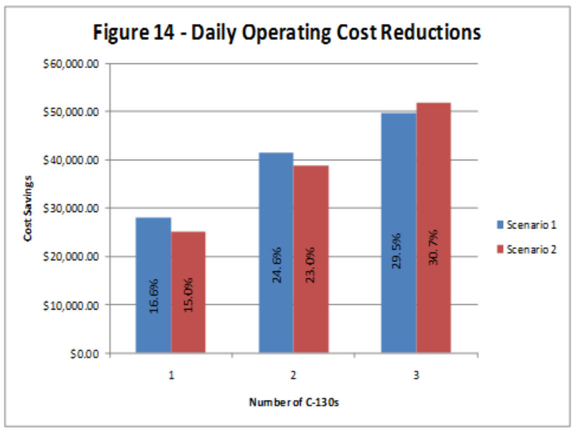


Figure 14 - Daily Operating Cost Reductions

Analysis of the results provided by the model demonstrates that significant CH-47 blade and airframe hours can be saved from even incorporating a single C-130 aircraft. Even though the established scenario here is small in comparison to the total number of potential bases that are serviceable in the Iraq and Afghanistan theaters, this scenario is on par with the size of any given daily workload network. Applying actual CH-47 and C-130 flight travel times between destinations in the theater network will still provide significant savings in CH-47 operating hours.

Model.

Keeping in focus with the research objectives of this research, those objectives are revisited in order to evaluate the model in meeting them. Our first objective is to determine how many CH-47 blade hours can be saved. Based on scenario and model capability demonstrations, it is possible to determine an amount of savings in relation to a benchmark involving only CH-47 aircraft. The term "an amount of savings" is used to emphasize that this model, depending on platform, has to be executed as a heuristic tool if the modeled network is quite large. From a practical standpoint, a feasible solution can be found within the first few minutes of model execution. Furthermore, a good estimate on CH-47 blade hour savings can be measured with this model given that any aircraft is forced to replace a portion of the CH-47's workload. However these savings may or may not come at increased costs. If a higher capacity yet more costly aircraft is utilized to replace a lighter portion of the CH-47 workload, then blade hour will still be saved, but the overall daily costs are likely to soar. If a heavier portion of the CH-47 workload is replaced, then both blade hour and cost savings will be realized.

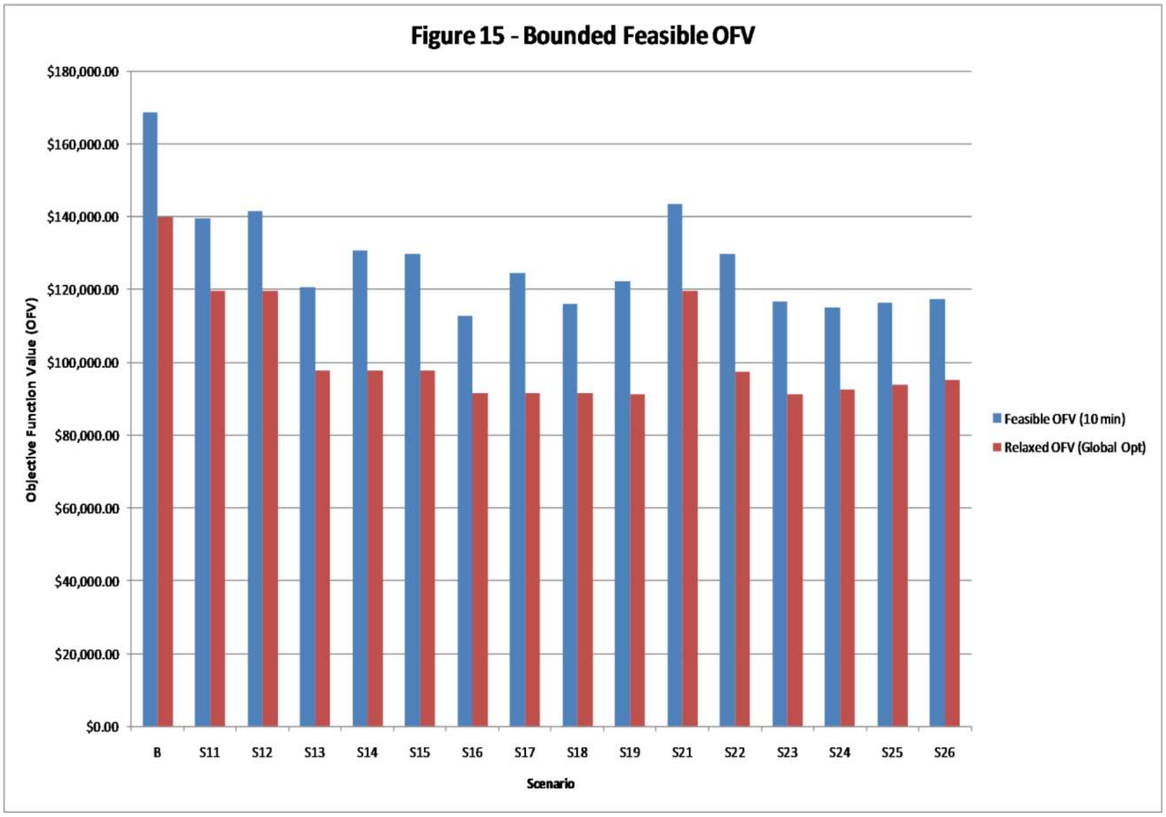
The second research objective is to determine what additional C-130 effort is required to garner CH-47 hour savings. This objective goes hand-in-hand with the first objective. A specific amount of blade hour savings is directly linked to both the number of C-130 aircraft utilized and what sorties those aircraft fly. The model developed in this research is capable of generating solutions that can aid in determining CH-47 savings. Additionally, the model is constructed with sufficient robustness that other types of aircraft may be inserted with some effort. However, stepping back and evaluating a larger picture of the CH-47 workload problem, what is not addressed is the effort

required to make C-130s available from a joint perspective. With that said, this model may be best utilized as a high-level planning tool rather than a potential daily sortie planner.

The model presented and executed during the course of this research is a realization of the third research objective. At the start of this research, it is decided that a method be constructed that determines the correct mix of C-130s and CH-47s based on cargo and passenger requirements and the number of C-130 capable and non-C-130 capable fields. This method has been realized in the form of an integer-linear program executed as a heuristic tool. For large networks, the best that this method can offer is a feasible solution found within minutes of model execution on a laptop. Permitted sufficient time and high-computing hardware, a global optimal solution can be potentially found. This may be viewed as a shortfall, but no other tool exists that can yield a practical solution (although not the best) and give the user significant robustness in model and network settings.

By relaxing the variable integer constraints, a global optimal solution can be obtained. However, this solution will not have practical merits in terms of variables as they will be fractured. But we do gain insight into a theoretical lower bound for the scenario as well as "how good" is the feasible solution observed after ten minutes of runtime. This theoretical lower bound must be equal to or less than the non-relaxed global optimal solution. On average, each feasible solution obtained from the scenarios is within eighty percent of the theoretical optimal solution. Inspection of each scenario's variable solution report showed fractured variables, which indicates that the global optimal solution found using relaxed variables is less than the theoretical global optimal

solution using non-relaxed variables. This suggests that the obtained feasible solution may be even closer to a global optimal solution than eighty percent. Figure 15 shows the obtained feasible solution using non-relaxed variables after ten minutes of runtime compared to a global optimal solution obtained using relaxed variable constraints.



In the above figure, “B” represents the baseline scenario and “S##” represents the various scenarios using the initial format “S#-#.”

V. Discussion

Relevance of the Current Investigation

Current nation-building and stabilization operations in Iraq and Afghanistan have, to date, proven to be the U.S.'s most enduring and cost intensive efforts. With a continued U.S. presence and a sustained need for supplies and transportation, it can be safe to anticipate a need for the CH-47 aircraft for years to come. As a result, USTRANSCOM needs to ensure that this air asset is not excessively utilized such that its lifespan is cut short. The investigative effort conducted here has been to ensure that the CH-47 continues to provide essential airlift capability as needed in future years.

The tool developed in this thesis provides essential capabilities relevant to the CH-47 workloads in Iraq and Afghanistan. Moreover, this tool can be applied as a heuristic to many other scenarios involving other types of aircraft. It has been demonstrated to provide practical results which can be applied to the various scenarios involving the CH-47 aircraft with augmentation by the C-130 in Iraq and Afghanistan.

Reflections on Obtained USTRANSCOM Data

Data provided by USTRANSCOM provided significant insight into the CH-47 operations in Iraq for 2009 and 2010. Much of the underlying network is derived based on where CH-47s conducted sorties. In order to avoid classification, notimal nodes with emulated aircraft travel times are utilized. To replicate the underlying CH-47 networks in Iraq, one would simply need to assign real air base names to nodes and enter actual aircraft flight times in the model's data matrices.

Since it was decided to construct a general model capable of capturing the CH-47 networks in Iraq and Afghanistan, no data is actually used to generate the network(s). The data required to construct exact CH-47 networks is inputted by the user, effectively mitigating the need to evaluate data integrity from the perspective of this research. However, this could become an issue for the user. The model generates the required variables and constraints based on the data entered by the user.

In essence, the data from USTRANSCOM is invaluable in determining the model's aircraft engineering parameters, flying hour costs, and network flow and accessibility. The data contained records of CH-47 flight sorties along with cargo and PAX taken on or dropped off. These records provided a great deal of insight into the utilization of the aircraft in the Iraq theater. This utilizations played a factor in determining how best to adjust the weight capacity of the aircraft to account for volume restrictions.

Perspective

From an employment perspective, the model is intended to minimize the cost in an established network given available vehicle resources while transiting all node requirements. At the most basic level, the mode attempts to find the best mix of vehicles given capacity and time constraints, and where those vehicles should travel. It is possible to utilize this model as a daily planning tool as it yields a feasible solution rapidly.

Evaluating the developed model from a mathematical perspective, it combines aspects of minimum cost flow and vehicle routing models on a hub-and-spoke network uniquely tailored to provide balanced solutions for the CH-47 workload. However, despite being designed with variables and parameters for the CH-47 and C-130 aircraft,

the model can be easily manipulated to represent any kind of vehicle flow. A major drawback is the number of variables required to track vehicle designations and route combinations that can rapidly explode.

On a researcher note, model development took several iterations and a significant amount of time to develop. The lengthy endeavor is primarily due to attempts to keep the model open-ended enough such that it is not limited to the exact CH-47 and C-130 networks currently employed in Iraq. Similarly, it is not desired to limit the model to those particular aircraft. The end result of this research is a general model in which the user is free to set their parameters, vehicles, and network. The only restrictions to the model in regards to being open-ended are that cross-flow between hubs is non-existent and all cargo requirements at the destinations must be transported to a hub.

Conclusion

Replacing any portion of the CH-47 workload with a larger capacity aircraft will, intuitively, net savings on the CH-47 airframe and blade hours. The developed model links the CH-47 workload replacement through the use of other aircraft types and generates a practical solution. Through investigative studies of network flow models, vehicle routing, and hub-and-spoke networks, this research has tied the areas together into a single mathematical program designed to reduce the CH-47 workload. This study concludes by offering an integer-linear program that can be used as a heuristic that attempts to discover the minimal amount of aircraft use required to move 100% of a set of cargo requirements, given a set of hubs and destinations that include network restriction.

Appendix A. LINGO-Based Model

```
MODEL:
TITLE Optimal CH-47 and C-130 Workload Balance;

!*******************************************************************;
!***Variables***;
!     SORTIE(T,H,N,I,J) = 1 if a vehicle formation of type T, number N,
                          stationed at hub H travels from node I to
                          node J;
!                         0 otherwise;

!                         where T = 1, 2, ..., 'user specified';
!                               N = 1, 2, ..., 'user specified';
!                               H = 1, 2, ..., 'user specified' and
                                    less than [max(I,J) - 1];
!                               I, J = 1, 2, ..., 'user specified';

!     CT(T,H,N,J) = amount of cargo taken from node J by a vehicle of
                    type T, number N, stationed at hub H;

!NOTE: CT refers to 'Cargo Taken' and cargo units are user-defined in
       the vehicle settings;

!     PT(T,H,N,J) = number of PAX taken from node J by a vehicle of
                    type T, number N, stationed at hub H;

!NOTE: PT refers to 'PAX Taken';

!     NUM_REQ(T) = the minimum number of type T vehicle formations
                   required to move all PAX/cargo requirements;

!     V_USED(T,H,N) = 1 if a vehicle formation of type T, number N was
                      used to conduct sorties out of hub H;
!                     0 otherwise;

!*******************************************************************;
SETS:
      DESTINATION: CARGO_REQ, PAX_REQ;
      HUB: AVAIL_CH47, AVAIL_C130;
      VEHICLE_TYPE: NUM_HUBS, COST, WT_CAP, MAX_CARGO_WT, MAX_PAX,
                    NUM_FORMATION, WT_CAP_ADJ, DURATION_LIMIT, NUM_REQ,
                    TOTAL_VEHICLE_TIME, TOTAL_VEHICLE_COST,
                    TOTAL_CARGO_MOVED, TOTAL_PAX_MOVED;
      VEHICLE_NUM;
      DxD(DESTINATION, DESTINATION): TIME_CH47, TIME_C130, SWITCH_CH47,
                                     SWITCH_C130;
      TxH(VEHICLE_TYPE, HUB): AVAIL;
      HxN(HUB, VEHICLE_NUM);
      TxHxN(VEHICLE_TYPE, HUB, VEHICLE_NUM): V_USED;
      TxDxD(VEHICLE_TYPE, DESTINATION, DESTINATION): TIME, SWITCH;
      HxNxD(HUB, VEHICLE_NUM, DESTINATION);
      TxHxNxD(VEHICLE_TYPE, HUB, VEHICLE_NUM, DESTINATION): CT, PT;
      HxNxDxD(HUB, VEHICLE_NUM, DESTINATION, DESTINATION);
```

```
      TxHxNxDxD(VEHICLE_TYPE, HUB, VEHICLE_NUM, DESTINATION,
                DESTINATION): SORTIE;
ENDSETS

!*********************************************************************;
DATA:
!***HUB AND DESTINATION SETTINGS***;
DESTINATION = N1 N2 N3 N4 N5 N6 N7 N8 N9 N10 N11 N12 N13 N14 N15;
!Sets total number of nodes (combined hubs and destinations) for the
network;
HUB = H1 H2 H3 H4 H5;
!Sets the maximum number of hubs for the network;

!***CARGO AND PAX SETTINGS***;
CARGO_REQ = @FILE('Cargo.PRN');
!This file contains all the cargo requirements (in 1000 lbs) for each
destination;
PAX_REQ = @FILE('PAX.PRN');
!This file contains all the PAX requirements (in 1000 lbs) for each
destination;

!***VEHICLE DATA***;
!******CH47******;
TIME_CH47 = @FILE('CH47_Times.PRN');
!These are the times in hours req'ed for a CH-47 to fly from node I to
node J;
SWITCH_CH47 = @FILE('CH47_Switch_Matrix.PRN');
!This binary matrix controls where a CH-47 can fly;
AVAIL_CH47 = @FILE('CH47_Availability.PRN');
!This file contains the number of CH-47 stationed at each node;

!******C130******;
TIME_C130 = @FILE('C130_Times.PRN');
!These are the times in hours req'ed for a C-130 to fly from node I to
node J;
SWITCH_C130 = @FILE('C130_Switch_Matrix.PRN');
!This binary matrix controls where a C-130 can fly;
AVAIL_C130 = @FILE('C130_Availability.PRN');
!This file contains the number of C-130 stationed at each node;

!***VEHICLE SETTINGS***;
VEHICLE_TYPE = CH47 C130;
!Enter a vehicle type ID designator here;
VEHICLE_NUM = A1 A2 A3 A4 A5 A6 A7;
!NOTE: This set must contain a number of vehicle slots equal or greater
       than the highest number of required vehicle for either type,
       i.e. {A1,A2,...,AN} where N = MAX(NUM_REQ(T1), NUM_REQ(T2),...);
NUM_HUBS = 5 2;
!Establishes the number of hubs in the network starting at node 1, node
2, ..., node J for each vehicle type;
COST = 2723 3650;
!Established the cost of vehicle operation per hour by type;
WT_CAP = 8.5 45.0;
!Sets the weight (in 1000 lbs) capacity for the CH-47 VEHICLE;
MAX_CARGO_WT = 8.5 45.0;
```

```
!Sets a maximum allocated amount of weight for cargo by vehicle type,
must be equal or less than WT_CAP;
MAX_PAX = 8 150;
!This is the maximum number of PAX that can be carried by vehicle type;
NUM_FORMATION = 2 1;
!Establishes the number of vehicles that travel together in a formation
or group by type;
WT_CAP_ADJ = 0.55 0.55;
!Adjusts a vehicle's weight capacity to account for the vehicle's
volume limits by type;
DURATION_LIMIT = 6 12;
!Sets the duration limit on how long a vehicle can continuously
operate;
WT_PER_PAX = 0.4;
!Sets the allocated weight (in 1000 lbs) for a single PAX;
ENDDATA

!NUM_REQ(1) = 17;
!NUM_REQ(2) = 0;
!These settings force a number of specific vehicle type to be used,
must be equal or less than the total number of a vehicle type
available;

!*********************************************************************;
!***PREPARATION OPERATIONS***;
!Used to read in vehicle travel time data;
@FOR(TxDxD(T,I,J)|T #EQ# 1:
      TIME(T,I,J) = TIME_CH47(I,J));

@FOR(TxDxD(T,I,J)|T #EQ# 2:
      TIME(T,I,J) = TIME_C130(I,J));

!Used to read in vehicle node access data;
@FOR(TxDxD(T,I,J)|T #EQ# 1:
      SWITCH(T,I,J) = SWITCH_CH47(I,J));

@FOR(TxDxD(T,I,J)|T #EQ# 2:
      SWITCH(T,I,J) = SWITCH_C130(I,J));

!Used to read in vehicle availability data;
@FOR(TxH(T,H)|T #EQ# 1:
      AVAIL(T,H) = AVAIL_CH47(H));

@FOR(TxH(T,H)|T #EQ# 2:
      AVAIL(T,H) = AVAIL_C130(H));

!*********************************************************************;
!***OBJECTIVE FUNCTION***;
MIN = @SUM(TxHxNxDxD(T,H,N,I,J)|I #NE# J:
      COST(T) * NUM_FORMATION(T) * TIME(T,I,J) * SORTIE(T,H,N,I,J));

!*********************************************************************;
!***CONSTRAINTS***;
!******Vehicle availability and requirement constraints******;
!*********Constraint 1a*********;
```

```
@SUM(VEHICLE_TYPE(T):
      NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) * NUM_REQ(T)) >=
@SUM(DESTINATION(J):
      CARGO_REQ(J) + WT_PER_PAX * PAX_REQ(J));

!*********Constraint 1b*********;
@SUM(VEHICLE_TYPE(T):
      NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) * NUM_REQ(T)) <=
      2 * @SUM(DESTINATION(J):
            CARGO_REQ(J) + WT_PER_PAX * PAX_REQ(J));

!*********Constraint 2a*********;
@FOR(VEHICLE_TYPE(T):
      @SUM(HxN(H,N): V_USED(T,H,N)) = NUM_REQ(T));

!*********Constraint 2b*********;
@FOR(TxH(T,H):
      @SUM(VEHICLE_NUM(N): V_USED(T,H,N)) <= AVAIL(T,H));

!******PAX/cargo vehicle capacity constraints******;
!*********Constraint 3*********;
@FOR(TxHxN(T,H,N):
      @SUM(DESTINATION(J):
            CT(T,H,N,J) + WT_PER_PAX * PT(T,H,N,J)) <=
            NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) *
            V_USED(T,H,N));

!*********Constraint 4*********;
@FOR(TxHxN(T,H,N):
      @SUM(DESTINATION(J):
            CT(T,H,N,J)) <=
            NUM_FORMATION(T) * MAX_CARGO_WT(T) * V_USED(T,H,N));

!*********Constraint 5*********;
@FOR(TxHxN(T,H,N):
      @SUM(DESTINATION(J):
            PT(T,H,N,J)) <=
            NUM_FORMATION(T) * MAX_PAX(T) * V_USED(T,H,N));

!*********Constraint 6*********;
@FOR(DESTINATION(J):
      @SUM(TxHxN(T,H,N): CT(T,H,N,J)) = CARGO_REQ(J));

!*********Constraint 7*********;
@FOR(DESTINATION(J):
      @SUM(TxHxN(T,H,N): PT(T,H,N,J)) = PAX_REQ(J));

!*********Constraint 8*********;
@FOR(TxHxNxD(T,H,N,J):
      CT(T,H,N,J) <=
      NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) *
      @SUM(DESTINATION(I)|I #NE# J:
            SORTIE(T,H,N,I,J)));
```

```
!*********Constraint 9*********;
@FOR(TxHxNxD(T,H,N,J):
      PT(T,H,N,J) <=
      NUM_FORMATION(T) * MAX_PAX(T) *
      @SUM(DESTINATION(I)|I #NE# J:
            SORTIE(T,H,N,I,J)));

!******Network constraints******;
!*********Constraints 10a, 10b, 10c, 10d, 10e, and 10f*********;
@FOR(TxHxN(T,H,N):
      @SUM(DESTINATION(J)|J #NE# H:
            SORTIE(T,H,N,H,J)) = V_USED(T,H,N);

      @FOR(DESTINATION(J)|J #NE# H:
            @SUM(DESTINATION(I)|I #NE# J: -SORTIE(T,H,N,I,J)) +
            @SUM(DESTINATION(I)|I #NE# J:  SORTIE(T,H,N,J,I)) = 0);

      @SUM(DESTINATION(J)|J #NE# H:
            -SORTIE(T,H,N,J,H)) = -V_USED(T,H,N);

      @FOR(DxD(I,J)|I #NE# J #AND# I #NE# H #AND# J #NE# H:
            SORTIE(T,H,N,I,J) + SORTIE(T,H,N,J,I) <= V_USED(T,H,N));

      @FOR(DxD(K,J)|K #NE# H #AND# J #NE# H:
                  @SUM(DESTINATION(I)|I #NE# H #AND# I #NE# J:
                        SORTIE(T,H,N,I,J) + SORTIE(T,H,N,J,I)) <=
                        3 - SORTIE(T,H,N,H,K) - SORTIE(T,H,N,K,H));

      @FOR(DxD(I,J)|I #NE# J:
            SORTIE(T,H,N,I,J) <= SWITCH(T,I,J)));

!*********Constraint 11*********;
@FOR(TxHxNxDxD(T,H,N,I,J)|I #EQ# J:
      SORTIE(T,H,N,I,J) = 0);

!******Hour constraints******;
!*********Constraint 12*********;
@FOR(TxHxN(T,H,N):
      @SUM(DxD(I,J)|J #NE# I:
            TIME(T,I,J) * SORTIE(T,H,N,I,J) * SWITCH(T,I,J)) <=
            DURATION_LIMIT(T));

!******Variable constraints******;
!*********Constraint 13*********;
@FOR(VEHICLE_TYPE(T):
      @GIN(NUM_REQ(T)));

@FOR(TxHxNxD(T,H,N,J):
      @GIN(PT(T,H,N,J)));

!*********Constraint 14*********;
@FOR(TxHxN(T,H,N):
      @BIN(V_USED(T,H,N)));
```

```
!*********Constraint 15*********;
@FOR(TxHxNxDxD(T,H,N,I,J)|J #NE# I:
      @BIN(SORTIE(T,H,N,I,J)));

!**********************************************************************;
!***ADD'L OUTPUT CALCULATIONS***;

!NOTE: To reduce model runtime, disable (comment-out) the below
calculations;

!Total weight-carrying capacity of entire vehicle fleet;
AVAIL_WT_CAP = @SUM(TxH(T,H):
                  NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) *
                  AVAIL(T,H));

!Weight-carrying capacity used for the established scenario (what the
model actually used);
USED_WT_CAP = @SUM(VEHICLE_TYPE(T):
                  NUM_FORMATION(T) * WT_CAP_ADJ(T) * WT_CAP(T) *
                  NUM_REQ(T));

!Weight-carrying capacity required for the established scenario (what
is actually needed);
REQ_WT_CAP = @SUM(DESTINATION(J):
                  CARGO_REQ(J) + WT_PER_PAX * PAX_REQ(J));

!Vehicle use efficiency (gauges model efficiency - the closer this
ratio is to 1.0, the more efficient);
VEHICLE_EFF = USED_WT_CAP / REQ_WT_CAP;

!Total vehicle travel time based on sorties conducted;
@FOR(VEHICLE_TYPE(T):
      TOTAL_VEHICLE_TIME(T) =
      @SUM(HxNxDxD(H,N,I,J)|J #NE# I:
            NUM_FORMATION(T) * TIME(T,I,J) * SORTIE(T,H,N,I,J)));

!Total vehicle costs based on sorties conducted;
@FOR(VEHICLE_TYPE(T):
      TOTAL_VEHICLE_COST(T) =
      @SUM(HxNxDxD(H,N,I,J)|J #NE# I:
            COST(T) * NUM_FORMATION(T) * TIME(T,I,J) *
            SORTIE(T,H,N,I,J)));

!Total cargo moved by vehicle type;
@FOR(VEHICLE_TYPE(T):
      TOTAL_CARGO_MOVED(T) = @SUM(HxNxD(H,N,J): CT(T,H,N,J)));

!Total number of PAX moved by vehicle type;
@FOR(VEHICLE_TYPE(T):
      TOTAL_PAX_MOVED(T) = @SUM(HxNxD(H,N,J): PT(T,H,N,J)));

!**********************************************************************;
END
```

Appendix B. Blue Dart

How can we reduce the CH-47 workload in the Iraq AOR while maintaining fiscal responsibility? The answer is to replace the heaviest portions of the CH-47 workload with few high-capacity airlift assets. Including even a single high-capacity aircraft, despite incurring steeper operation costs, is significantly more efficient than the collective operation costs of the CH-47s replaced.

Current Iraq and Afghanistan internal supply and delivery operations are handled largely by the CH-47 aircraft. These operations consist of movement of cargo and passengers out of centralized hubs to various forward operating bases, and vice-versa. A subset of hubs and FOBs include overlapping operations performed by other airlift assets, namely the C-130. Through optimization modeling, significant overlap can be reduced by transferring a portion of the CH-47 workload to the C-130. This ultimately will increase the CH-47 lifespan and reduce daily operating costs.

Intuitively, replacing any portion of the CH-47's daily workload will net savings on the airframe, blade hours, and maintenance. However, this is will not necessarily translate into reduction of daily operation costs. We can better ensure costs are trimmed by targeting the heavy workload portions. An optimization model helps us find these targeted workloads.

A mixed-integer linear program has been developed to find a balance in the CH-47 workload with C-130 augmentation while minimizing daily operating costs. This model permits 'user-defined' network structures with which to evaluate costs utilizing a set of airlift assets. Currently, the model captures "pick-up and return to hub" cargo and

passenger scenarios. This capability mimics eighty percent of current CH-47 operations in Iraq and Afghanistan.

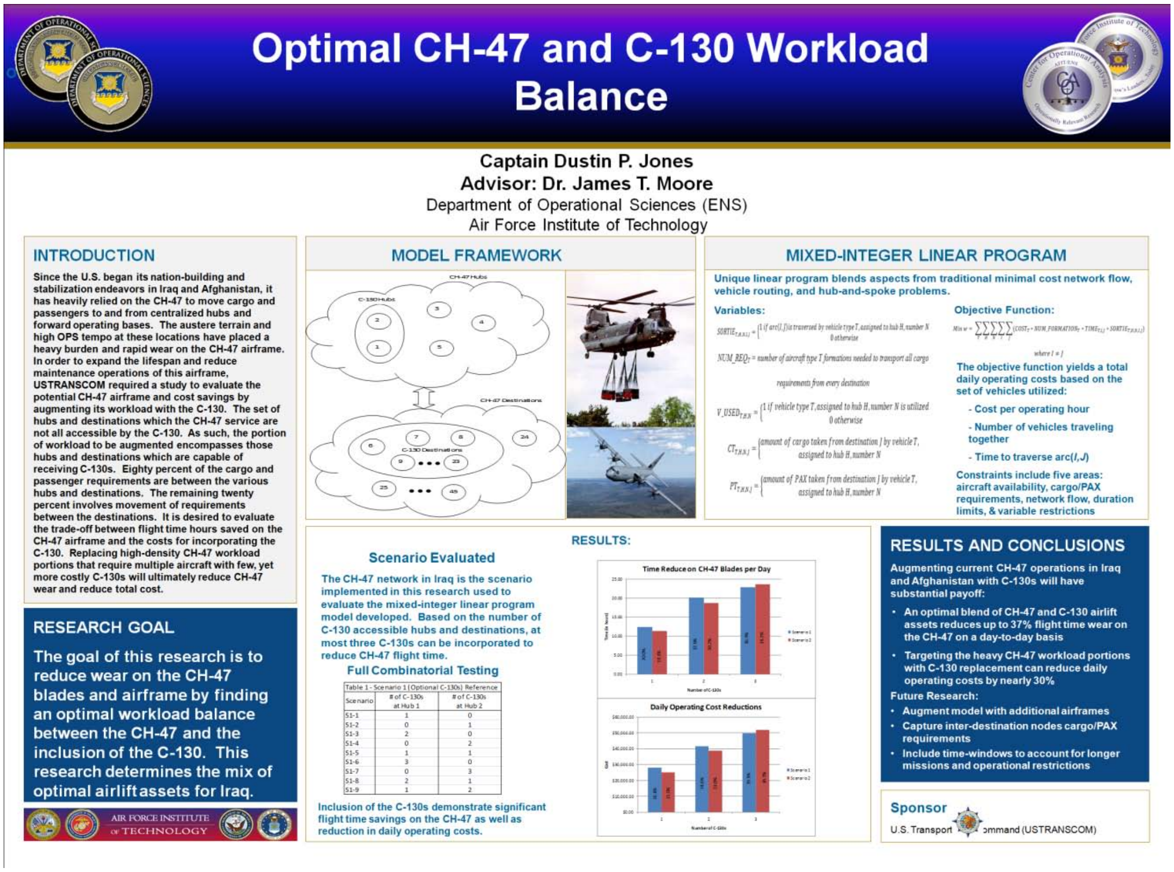
The CH-47 scenario in Iraq has been implemented in the model to demonstrate the potential value to the U.S. military. In this scenario, only a few hubs and FOBs have the capability to accommodate the C-130 aircraft. This implication suggests that the most CH-47 workload replaced by the C-130 consist of locations that are “C-130 capable.” Several scenarios were modeled, with each scenario relating to a higher incremented number of C-130s utilized. The savings are rather significant. Based on simulated cargo and passenger requirements for a given day, potentially between twenty-six and forty-three percent in CH-47 flight time can be saved in relation to the time that would be required if operations were strictly handled by the CH-47. Similarly, daily operating costs can potentially be reduced between twenty-five and thirty-six percent.

The results of this simulated scenario in Iraq attest to the potential value that optimization modeling can provide when applied to operations containing a high number of low capacity airlift assets. As the current U.S. nation-building and stabilization endeavors in Iraq and Afghanistan are sure to be long, the day-to-day savings garnered by the MILP employed will prove significant for the duration. On a similar note, the U.S. will need to ensure that the lifespan of its current airlift assets are lengthened. Additionally, suppose one desires to manage a large number of high capacity airlift assets more efficiently. Optimization modeling can yield similar results as well by replacing the lighter workload portions often conducted by high capacity aircraft with a more tailored capacity airlift solution.

The bottom line is: we must continue to meet our mission requirements under continually tightening fiscal constraints. Targeting the appropriate workload densities with the correctly tailored airlift assets will ensure continued mission success and cut down on daily costs. Optimization modeling ensures we have the right mix of airlift assets for the mission, operate at lower costs, and preserves are airframes.

Mar 11

Appendix C. Storyboard



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Vita

Captain Dustin P. Jones graduated from Mount Dora High School in Mount Dora, Florida. He entered undergraduate studies at the University of Central Florida in Orlando, Florida where he graduated with a Bachelor of Science degree in Aerospace Engineering in May 2005. He was commissioned through the Detachment 159 AFROTC at the University of Central Florida where he was nominated for a Regular Commission.

His first assignment was at Kirtland AFB as an operations analyst in Headquarters Air Force Test and Evaluation Center in July 2005. During this tour, he accomplished courses via correspondence with Touro University International earning a Masters in Business Administration with emphasis in Information Technology. In September 2009, he entered the Graduate School of Engineering and Management, Air Force Institute of Technology. Upon graduation, he will be assigned to the Pentagon.

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14. ABSTRACT
This study looks at the involvement of the C-130 in CH-47 airlift operations to reduce CH-47 usage and increase supply efficiency. The research focus is narrowed to current airlift operations in Afghanistan and Iraq in the CENTCOM theater of operation. A mathematical representation of current CH-47 operations augmented with C-130s is the foundation of this research. Particularly, these operations in CENTCOM's area of operations are formulated as linear transportation problems using network mathematics.

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